

# Coupled Simulation of Transport Phenomena in Directionally Solidifying $\gamma$ (TiAl) Alloy under Electromagnetic Fields

The Transport Phenomena here are described in term of Temperature, Melt Flow Velocity, Pressure, Solid-Volume Fraction and Concentration Fields *etc*

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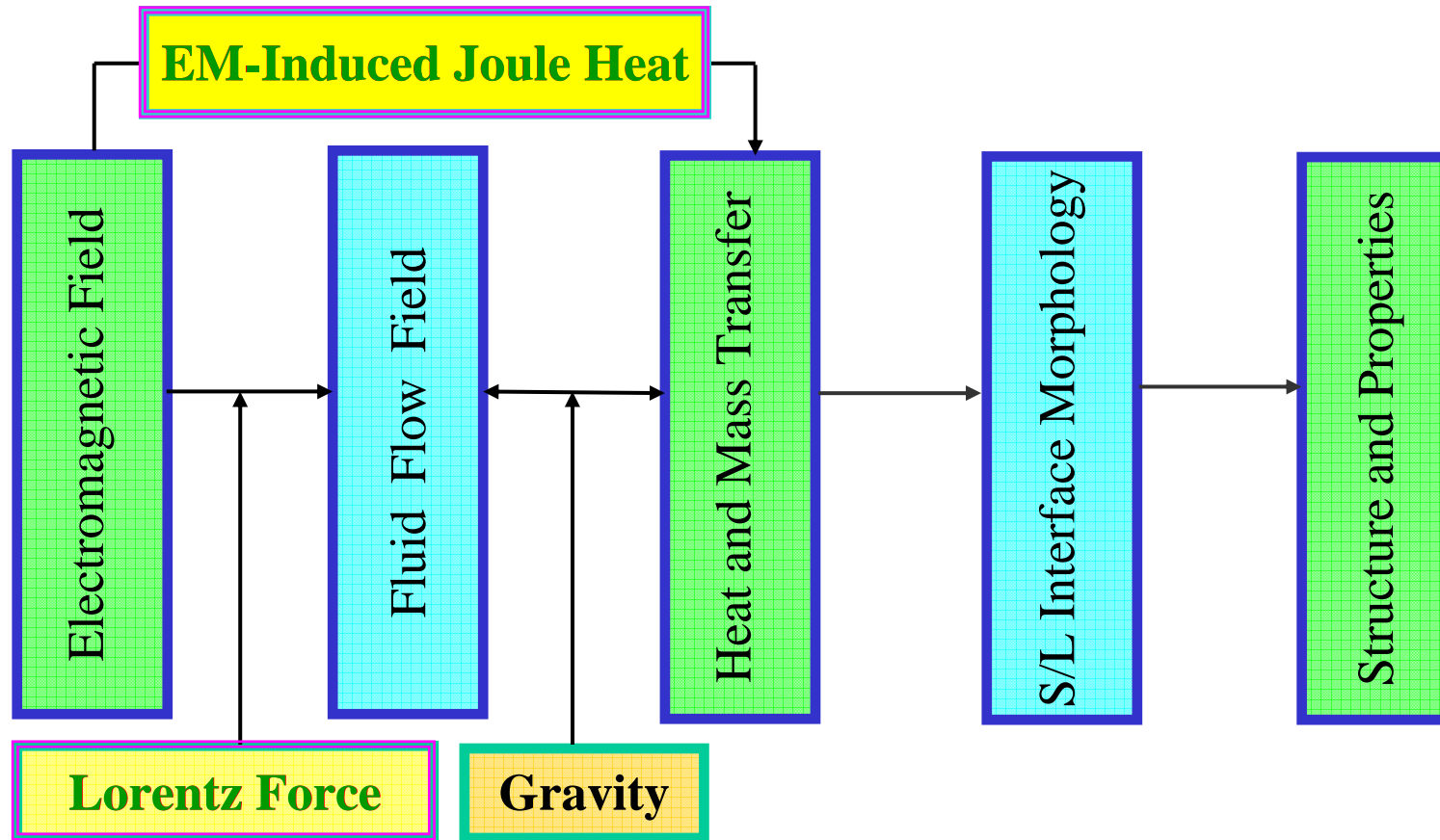
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# *INTRODUCTION*

- Heat energy and species mass transfer, as well as melt flow are basic transport phenomena in solidification processes of various alloy ingots and castings (called “*Solidification Transport Phenomena (STP)*”).
- These phenomena, in turn, play a critical role in controlling the solidification behaviors, structures and various defects formations, such as macrosegregation and porosity *etc.*

- In order to improve the qualities of the solidified alloy ingots/materials or to develop new materials processing methods, various electromagnetic (**EM**) **techniques** have been applied to materials solidification processes.
- Such as: Using ***alternating EM inductors*** or ***EM stirrers/brakes in continuous casting*** of Al alloy/steel ingots for melt flow/meniscus shape controls, or using an ***EM-induced cold-crucible technique*** to obtain directionally solidified  $\gamma(\text{TiAl})$ -base alloy ingots/materials (currently studied in the authors' research group).

- In such *EM materials processing* applications, the EM-induced solidification heat/mass transfer, and especially the melt flow behaviors can be *highly enhanced* (or *suppressed* in the case of a static magnetic field applied), through the *Lorentz forces* and *Joule heats*, and might be in a much more complex pattern.



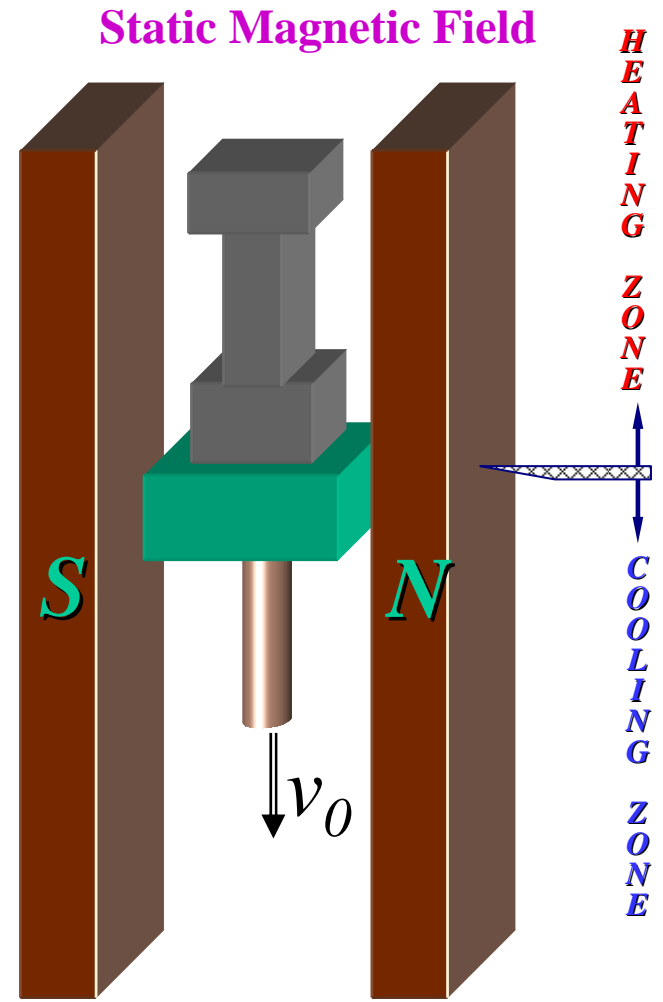
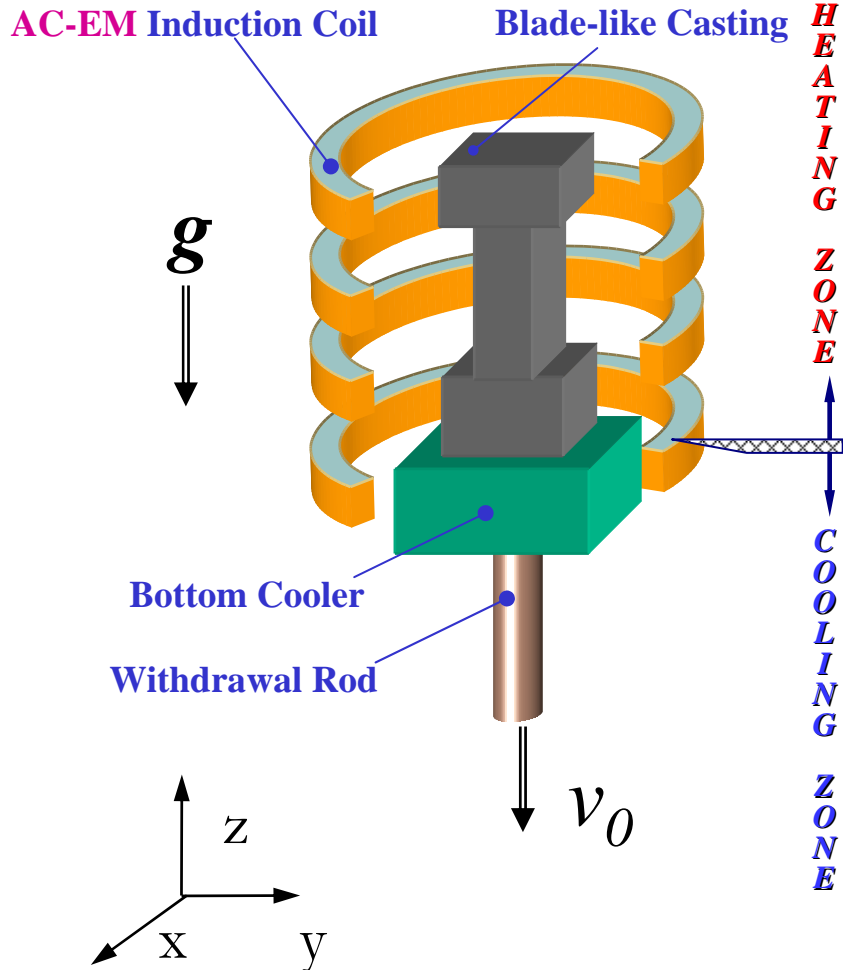
# *The Aim of This Presentation*

**To present:** An efficient multifield-coupled modeling method for the STP in alloy ingots/materials under **an arbitrary EM-field**, jointly using two different Computer Codes (One is **FEM-Based/Commercial**, another is **FDM-based/Authors'**).

## **The Main Tasks:**

- 1)** To further extend a previous continuum solidification-model and the corresponding numerical solution algorithms(including an extended **Direct-SIMPLE scheme** for directly solving the EM-/gravity-/solidification-shrinkage-driven and strongly pressure-linked flow problems) **to a more general EM-STP-case**;
- 2)** To propose an efficient technique for FEM→FDM data-conversion;
- 3)** Try to apply the modeling to EM-directional solidification of Ti alloy castings/ingots.

# Physical Model for EM-Directional Solidification



# *Electromagnetic solidification model*

- **A Continuum Model** (based on **Mixture Average Theory**) with the following assumptions:
  - 1) the external forces involved in a solidifying system are gravity and Lorentz force;
  - 2) no pores will occur, i.e. the geometric continuity,  $f_L + f_S = 1$ , holds for any region in the casting/ingot domain;
  - 3) the solid phase is macroscopically static during solidification;
  - 4) local thermodynamic equilibrium holds at the microscopic solid-liquid interfaces
  - 5) Newtonian and laminar liquid flow present;
  - 6) the model alloy is a binary system.

- *Solidification heat energy transfer*

$$\partial[(\rho c_p)_m T]/\partial t + \nabla(f_L \rho_L c_{pL} \mathbf{V} T) = \nabla[\lambda_m \nabla T] + \rho_S h(\partial f_S/\partial t) + q_J \quad (1)$$

where, the electromagnetically inducted Joule heat is given by

$$q_J = \mathbf{J}_G \mathbf{E} = \mathbf{J}_G^2/\sigma \quad (2)$$

- *Solidification solute mass transfer*

$$\partial(\rho C)_m/\partial t + \nabla(f_L \rho_L \mathbf{V} C_L) = \nabla[\mathbf{D}_L \nabla(f_L \rho_L C_L) + \mathbf{D}_S \nabla(f_S \rho_S C_S)] \quad (3)$$

where:

$$\partial(\rho C)_m/\partial t = (\rho_S C_S)^* \partial f_S/\partial t + \Phi f_S \partial(\rho_S C_S)^*/\partial t + (\rho_L C_L) \partial f_L/\partial t + f_L \partial(\rho_L C_L)/\partial t \quad (4)$$

$$\Phi = \theta \varphi / (1 + \theta \varphi), \quad \varphi = (D_S(T)/R_f) \zeta \cdot \mathbf{A}_{2N} \quad \text{and} \quad \theta = k(1 + \beta) f_S / f_L^2$$

- *L-S phase-change characteristic function for a specific alloy*

$$\mathbf{T}_{Liq} = \mathbf{T}_{Liq}(C_L^*) \quad (5)$$



- *Solidification mass conservation:*

$$\partial \rho_m / \partial t = -\nabla(\mathbf{f}_L \rho_L \mathbf{V}) \quad (6)$$

where,  $\partial \rho_m / \partial t \approx \rho_S^* \partial f_S / \partial t + \Phi f_S \partial \rho_S^* / \partial t + \rho_L \partial f_L / \partial t + f_L \partial \rho_L / \partial t$  (7)

- *Momentum transfer for bulk/interdendritic liquid flow:*

$$\partial(\mathbf{f}_L \rho_L \mathbf{V}) / \partial t + \nabla[(\mathbf{f}_L \rho_L \mathbf{V}) \cdot \mathbf{V}] = \nabla[\mu \nabla(\mathbf{f}_L \mathbf{V})] - \nabla(\mathbf{f}_L \mathbf{P}) - (\mu \mathbf{f}_L^2 / \mathbf{K}) \mathbf{V} + \mathbf{F}_B \quad (8)$$

For the present modeling, the body force term induced by external fields includes the gravity and Lorentz force:

$$\mathbf{F}_B = \mathbf{f}_L \rho_L \mathbf{g} + \mathbf{F}_L \quad (9)$$

the Lorentz force acting on the moving liquid phase during solidification:

$$\mathbf{F}_L = \sigma \mathbf{f}_L (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \times \mathbf{B} = \mathbf{f}_L \{ \mathbf{J}_G \times \mathbf{B} + \sigma [(\mathbf{V} \cdot \mathbf{B}) \mathbf{B} - \mathbf{B}^2 \mathbf{V}] \} \quad (10)$$

- *Maxwell's equations*

$$\nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D} / \partial t \quad (\text{Law of Maxwell-Ampere}) \quad (11)$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \quad (\text{EM-Induction Law of Faraday}) \quad (12)$$

$$\nabla \cdot \mathbf{D} = \rho_e \quad (\text{Law of Gauss}) \quad (13)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{Continuity of Magnetic Flux}) \quad (14)$$

- *Constitutive equations for the involved EM-medium materials*

$$\mathbf{D} = \epsilon \mathbf{E} \quad (\text{Constitutive Relationship of Electric Properties}) \quad (15)$$

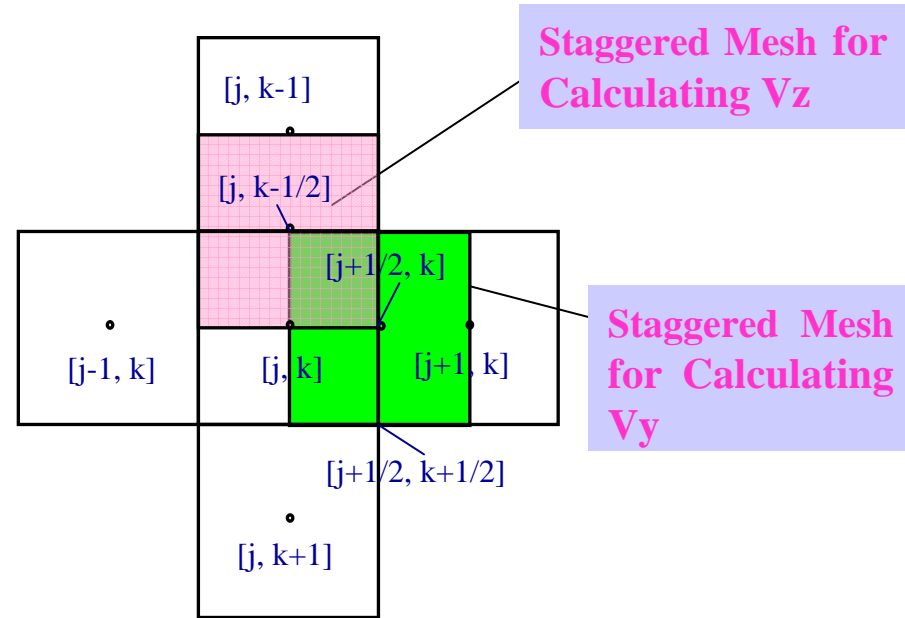
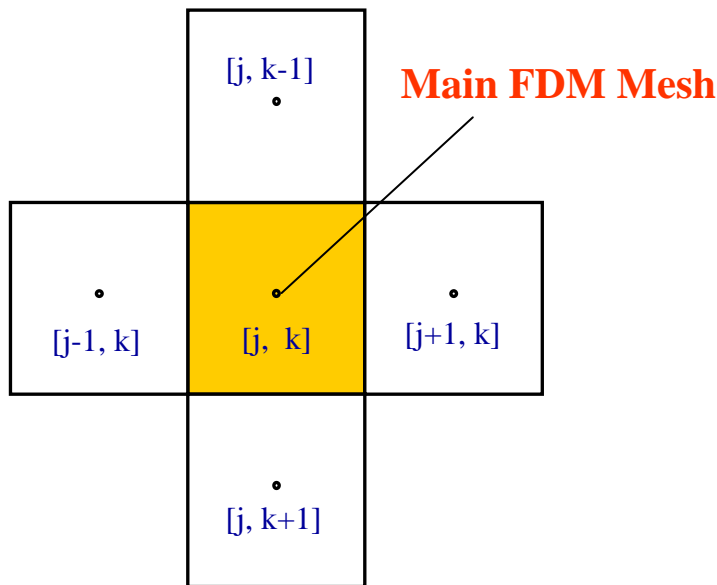
$$\mathbf{B} = \mu \mathbf{H} \quad (\text{Constitutive Relationship of Magnetic Properties}) \quad (16)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{f}_L \mathbf{V} \times \mathbf{B}) \quad (\text{Ohm Law for the Moving Metallic Melt}) \quad (17)$$

## *Model Numerical Solution Strategy*

- **Electromagnetic Fields** ( $B$  and  $J$  vector fields *etc*)  
—— Solved by **ANSYS 6.1** in a **FEM**-Scheme;
- **Solidification Transport Phenomena Equations** (scalar  $T$ ,  $f_S$ ,  $C_L$ ,  $P$  and vector  $V$  fields)  
—— Solved by a **FDM**-Scheme-based  
Computer Simulation Code  
(*developed by the present authors*)

# Numerical Solution Methods to the Equations for Alloy Solidification Transport Processes

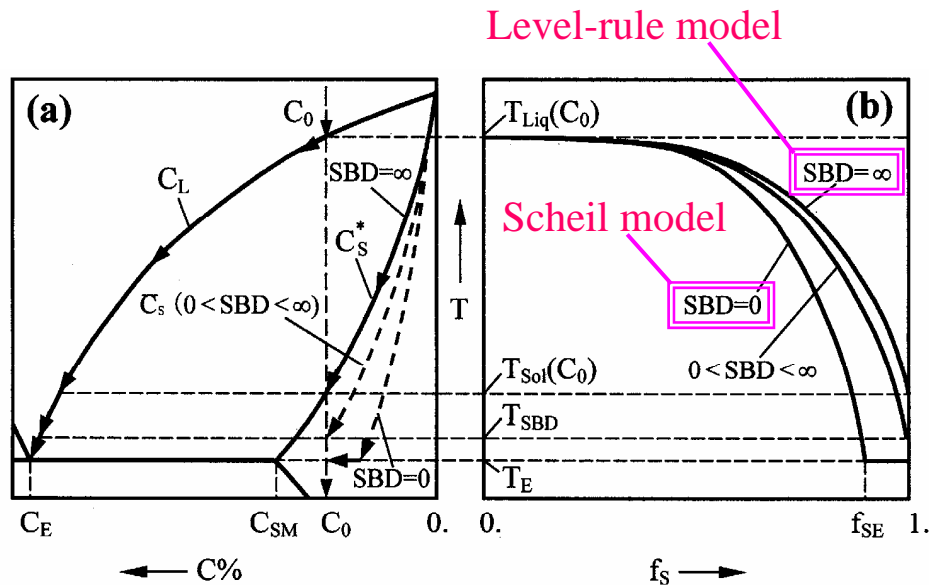


Main FDM Mesh  $[j, k]$  for Computations of the Scalar Variables  $T$ ,  $f_s$ ,  $C_L$  &  $P$

Staggered Meshes  $[j+1/2, k]$  and  $[j, k-1/2]$  for Computations of the Vectorial Variables  $V_y$  &  $V_z$

# Numerical Solution Methods to the Equations for Alloy Solidification Transport Processes

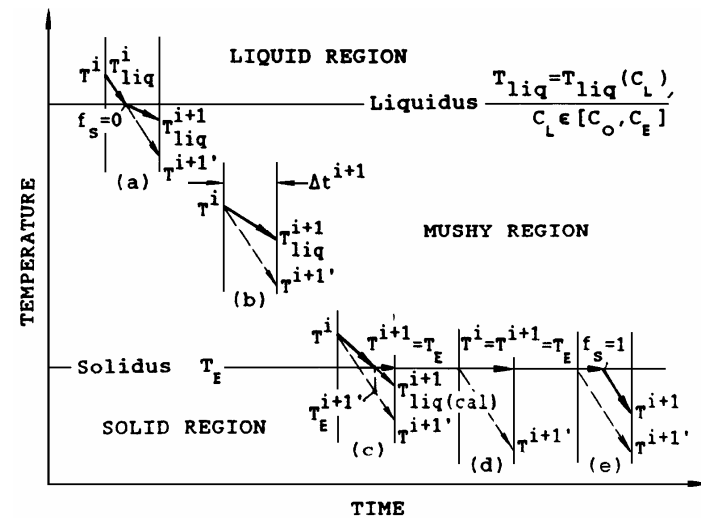
- Iterative Computations for the **Nonlinear and Strong  $T-f_S-C_L$  Coupling**



**Figure 1** Solidification behaviors of a binary alloy of nominal composition  $C_0 < C_{SM}$  and with different solid-back diffusion (SBD) extents (Thermodynamic equilibrium holds at the micro-scale S/L interface). (a) A portion of binary alloy phase diagram with the solidification paths of  $C_L$ ,  $C_S^*$  and  $\bar{C}_S$  for different SBD; (b) The corresponding curves of solid fraction versus liquidus.

Available for **ANY** Solid Back-Diffusion

Available for **BOTH** Single-Phase and Eutectic Solidification Portions



**Fig. 4** Revised temperature by the latent heat release  $T^{i+1}$  (solid line) and the estimated temperature by Eq. (8)  $T^{i+1'}$  (dashed line) for a control volume at the different nonequilibrium solidification stages for a binary eutectic system: (a)  $f_s^i = 0$ ,  $T^{i+1'} \leq T_{liq}(C_0)$ ; (b)  $f_{SE} > f_s^i > 0$ ,  $T_{liq}(C_0) > T^{i+1'} \geq T_E$ ; (c)  $0 < f_s^i < f_{SE}$  ( $T^i > T_E$ ),  $T^{i+1'} < T_E$ ; (d)  $1 > f_s^i \geq f_{SE}$  ( $T^i = T_E$ ),  $T^{i+1'} < T_E$ ,  $f_s^{i+1} < 1$ ; and (e)  $1 > f_s^i > f_{SE}$ ,  $T^{i+1'} < T_E$ ,  $f_s^{i+1} = 1$ .

(From: Daming Xu and Qingchun Li, "Numerical Method for Solution of Strongly Coupled Binary Alloy Solidification Problems", *Numerical Heat Transfer, Part A*, 1991, Vol.20, 181-201.)

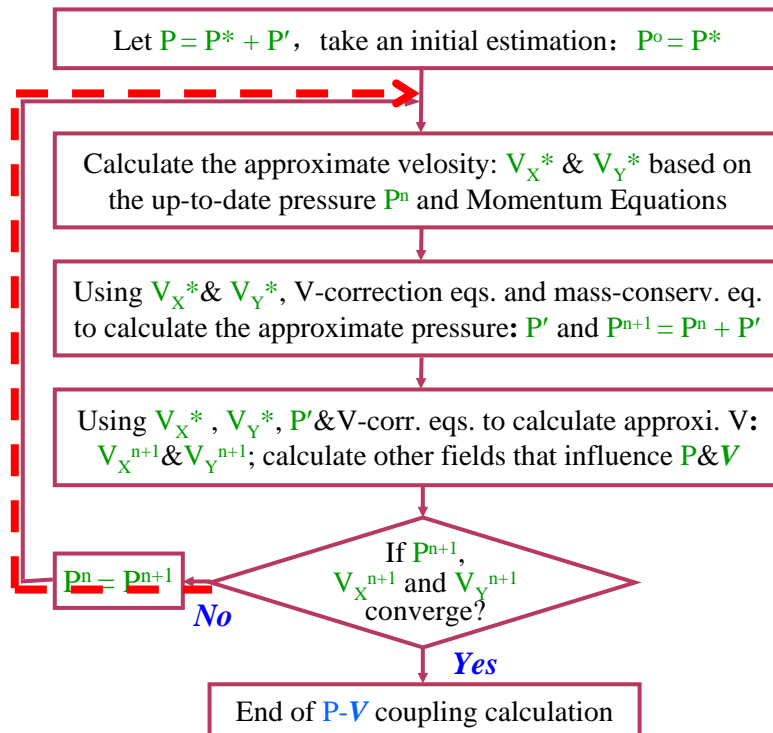
## *Numerical Solution Methods to the Equations for Alloy Solidification Transport Processes*

- Calculations of the Nonlinear and Strong **Pressure–Velocity (P–V) Coupling** in a Solidification Transport Process of Alloy Casting/Ingot **Takes Most Portion (>90%)** of the Entire Computational Efforts for the STP Numerical Simulation;
- Therefore, it is of Great Importance to Adopt a **Algorithm of High Computational Efficiency** for a Practical STP-Based Computer Simulation for the Solidification Processes of Alloy Castings/Ingots.

# Numerical Solution Methods to the Equations for Alloy Solidification Transport Processes

## Numerical Computations of the Nonlinear and Strong P-V Coupling

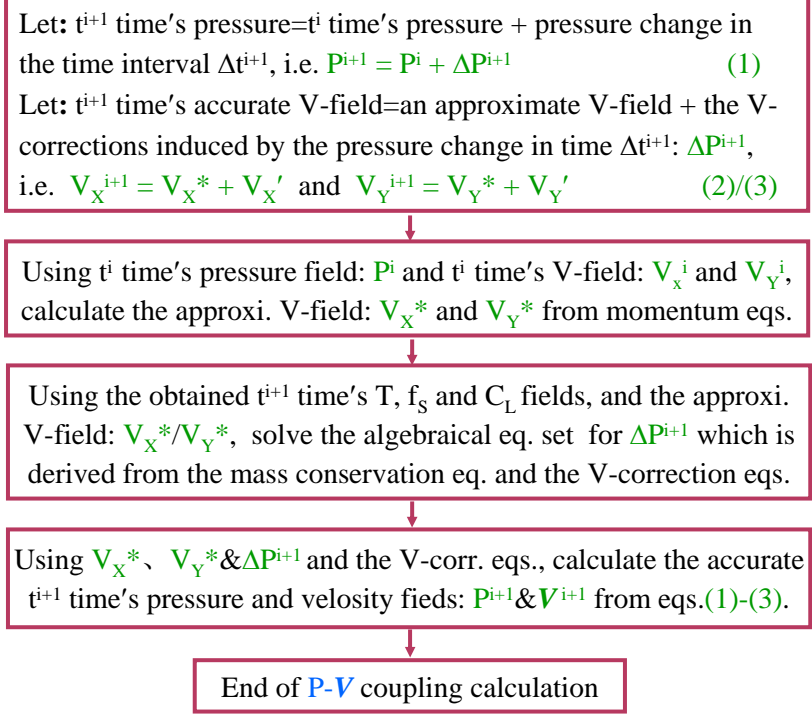
### SIMPLE Scheme widely used for Fluid Transport Systems by Subas V. Patankar



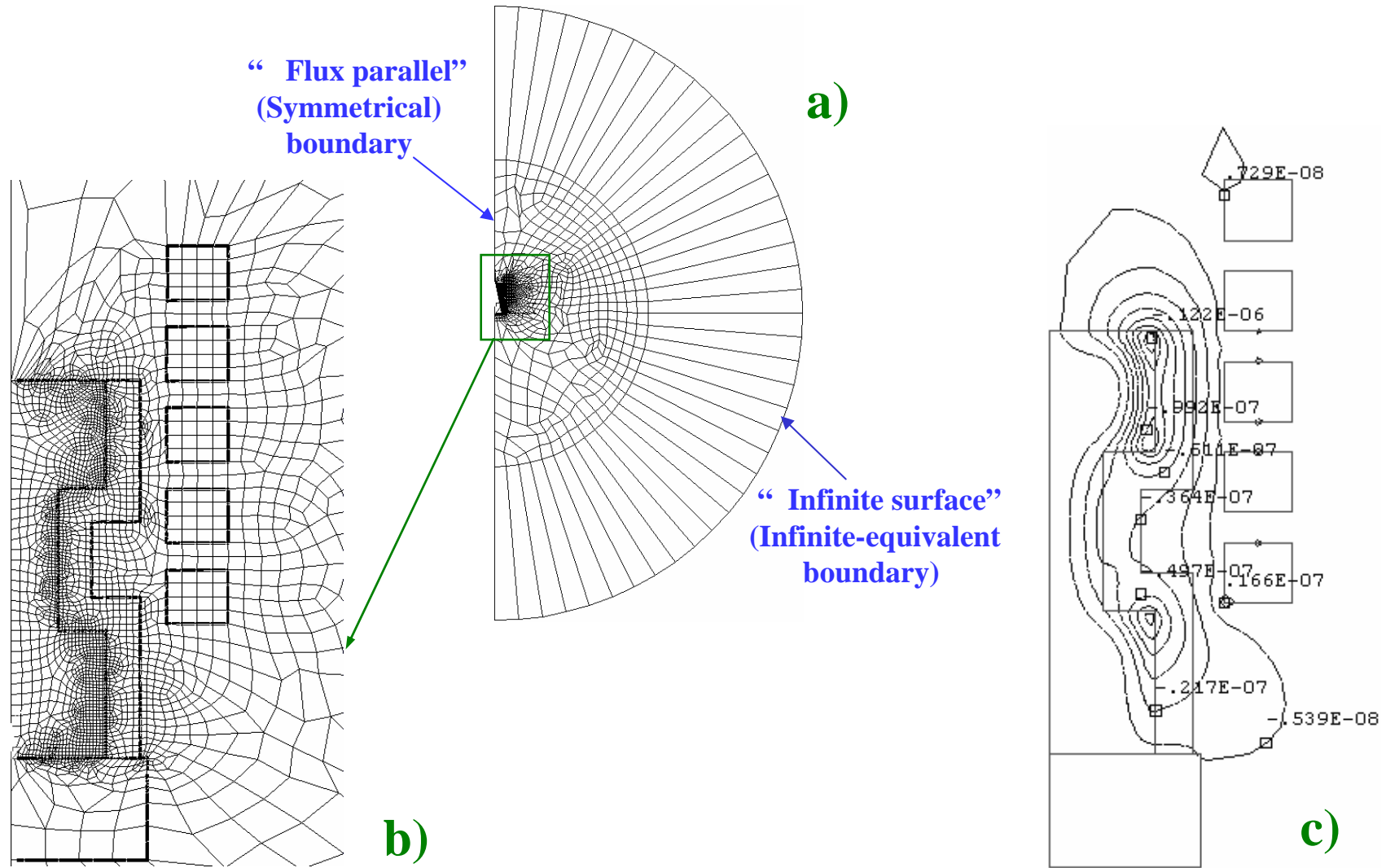
(From: Subas V. Patankar, *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill, 1980)

### Direct-SIMPLE Scheme by present Authors

(No Iteration Needed → C. Efficiency ↑↑ Available both for P-V Coupling in Pure Fluid Systems and Alloy Solidification Systems)



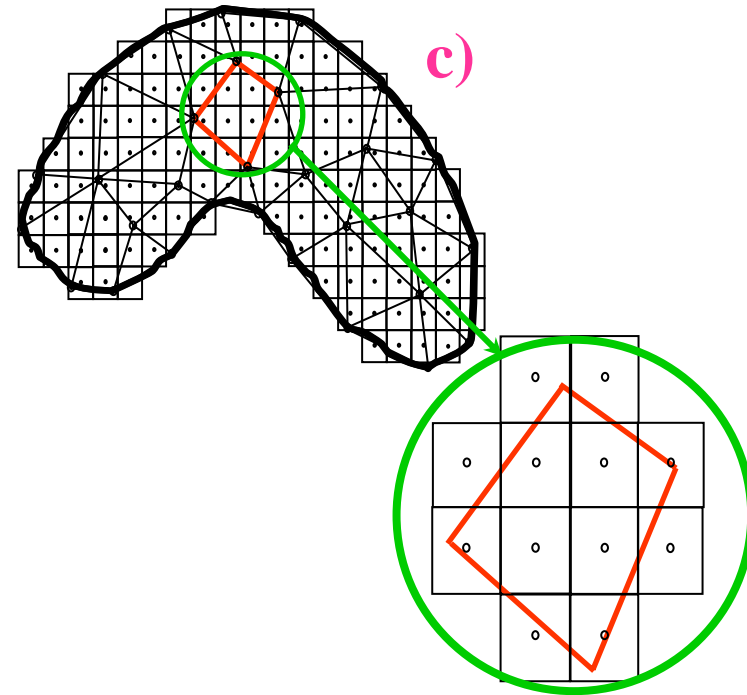
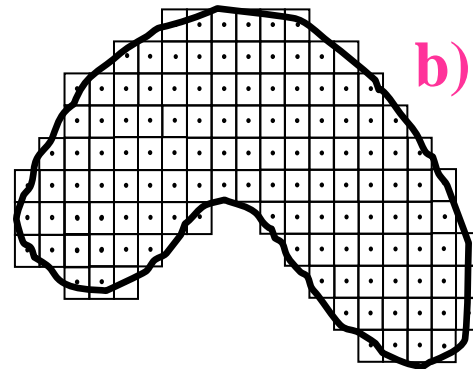
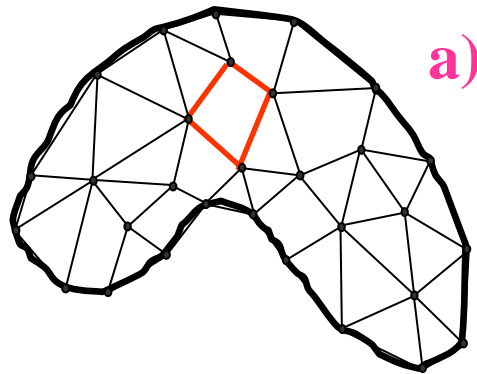
(From: Daming Xu and Qingchun Li, "Gravity- and Solidification-Shrinkage-Induced Liquid Flow in a Horizontally Solidified Alloy Ingot", *Numerical Heat Transfer, Part A*, 1991, Vol.20, 203-221.)



**FEM**-calculated results from **ANSYS 6.1** (TiAl casting, CaO mould, under a harmonic EM-field of 20kHz and with loads of 10kAt. The shaped casting has moved down for 40mm relative to the coils): **a)** meshed pattern for the whole EM-directional solidification system (right-half, with the EM-boundary definitions); **b)** the local meshed element grids around the casting; **c)** calculated distribution of the Az magnetic potential contours.

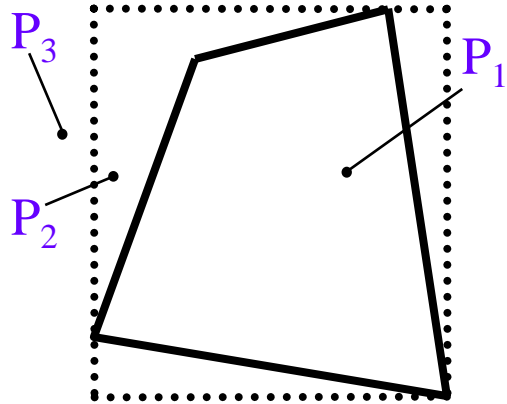


# *FEM-EM Results → FDM Format Conversion*

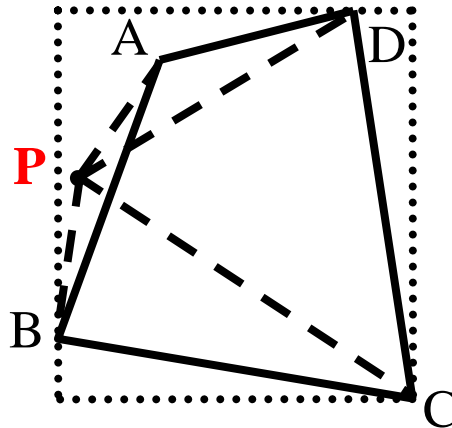


- a) A schematic FEM mesh pattern for a casting domain;
- b) A schematic FDM mesh pattern for the same casting domain;
- c) Overlap of a) and b)

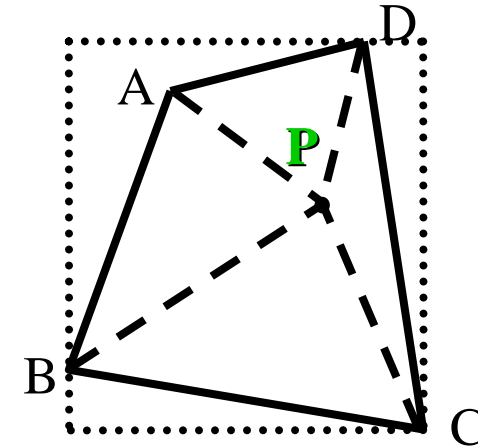
# FEM-EM Results → FDM Format Conversion



Maximum boundary of a FEM element (dot lines )



a)



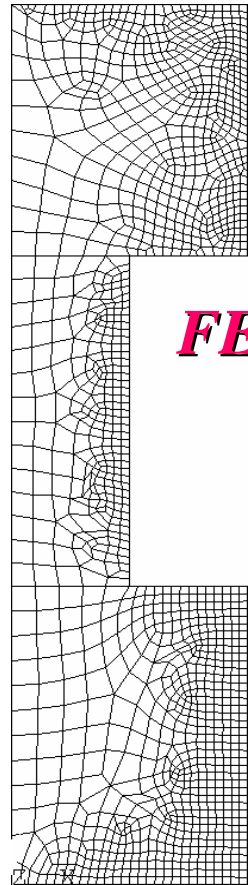
b)

Method to judge if a FDM node P lies inside the FEM element ABCD.

## Judgment:

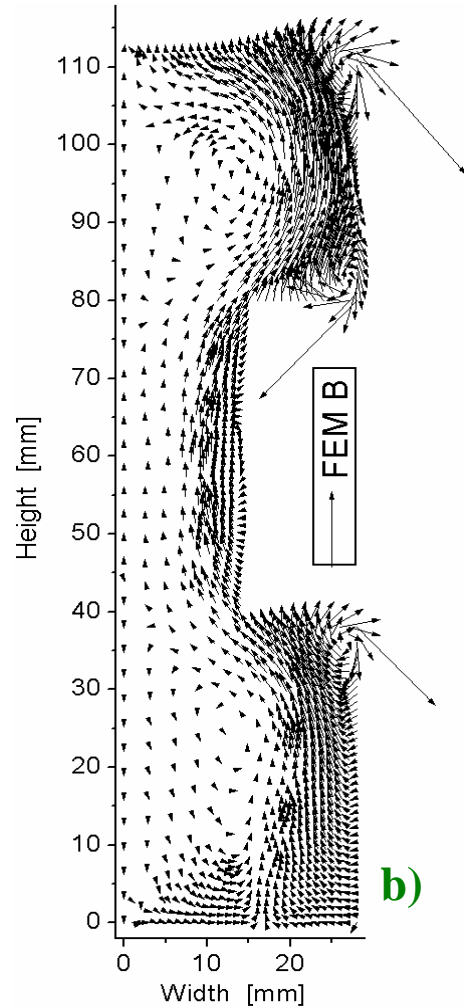
**IF**  $[S (= S_{ADP} + S_{DCP} + S_{CBP} + S_{ABP}) = S_{ADCB}]$ , **THEN** the FDM-node **P** is inside FEM-element ADCB; **ELSE**, the node **P** is outside the FEM-element.

# FEM-EM Results → FDM Format Conversion

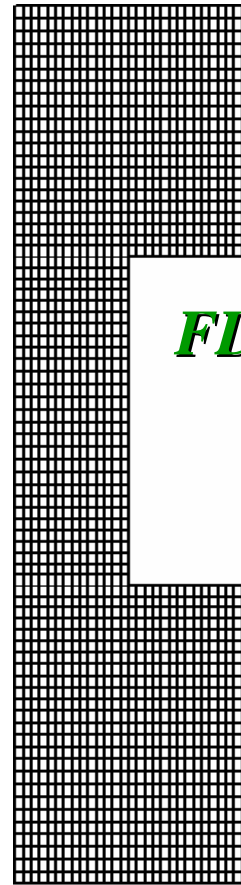


**FEM**

**a)**

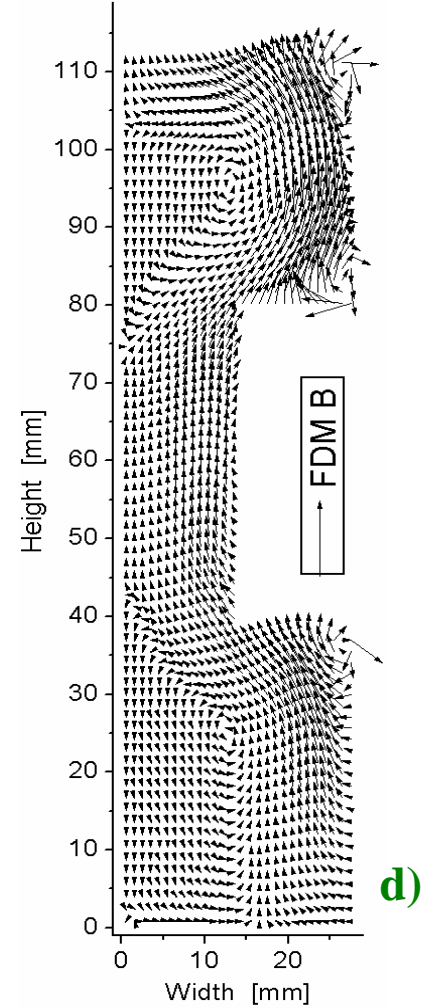


**b)**



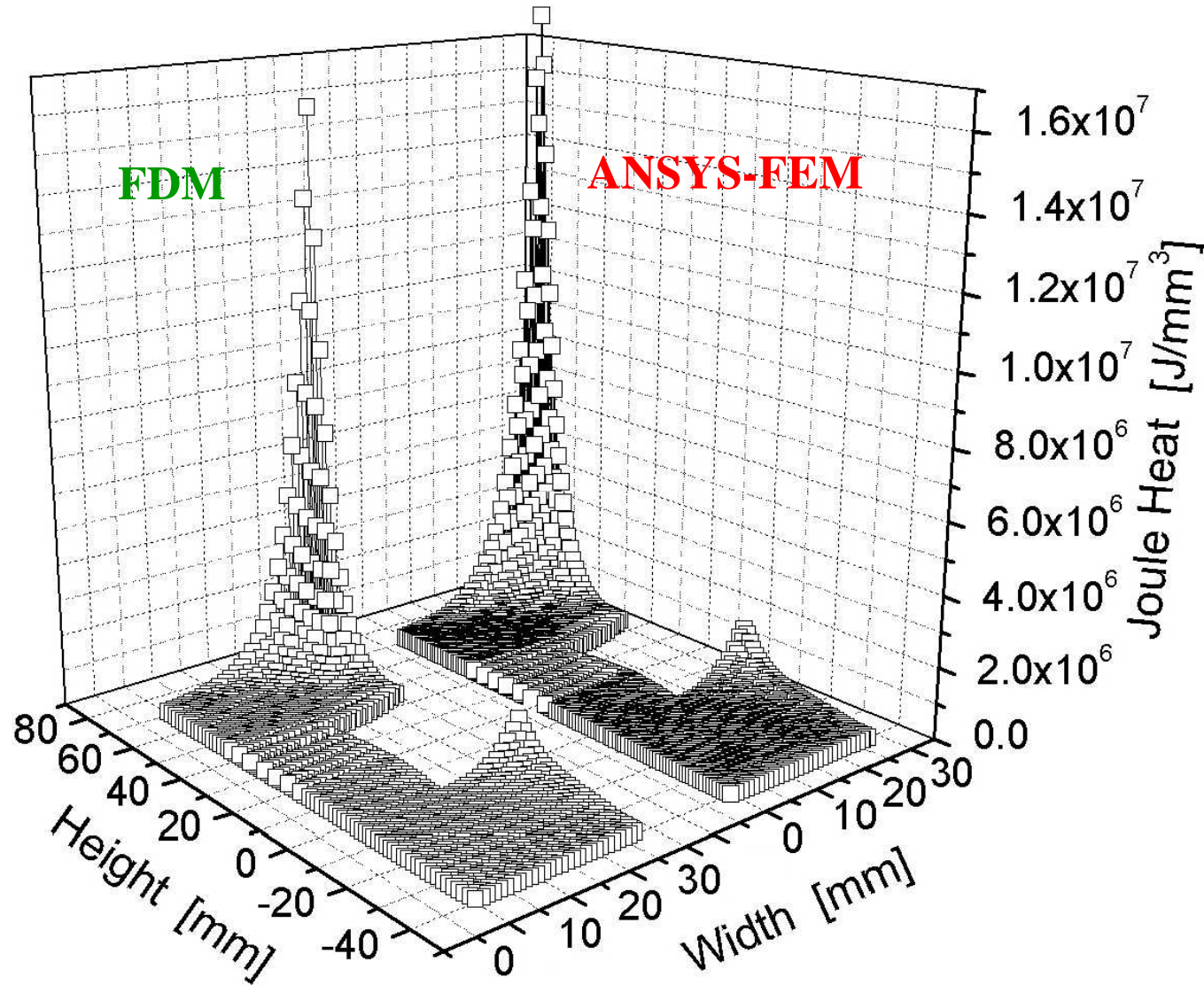
**FDM**

**c)**

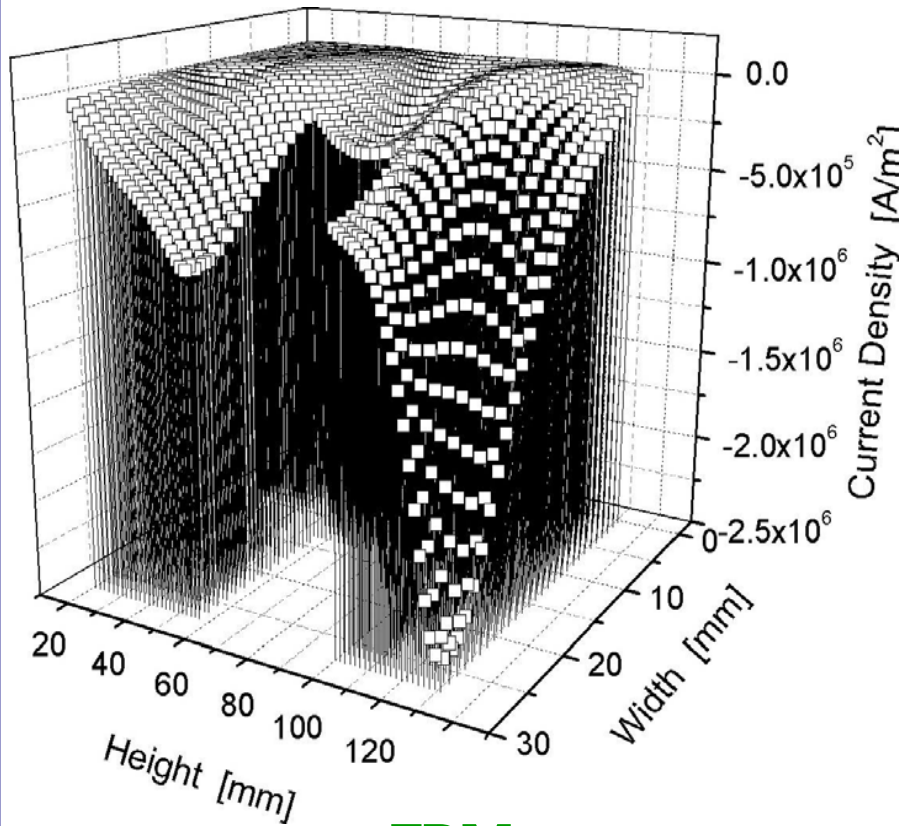


**d)**

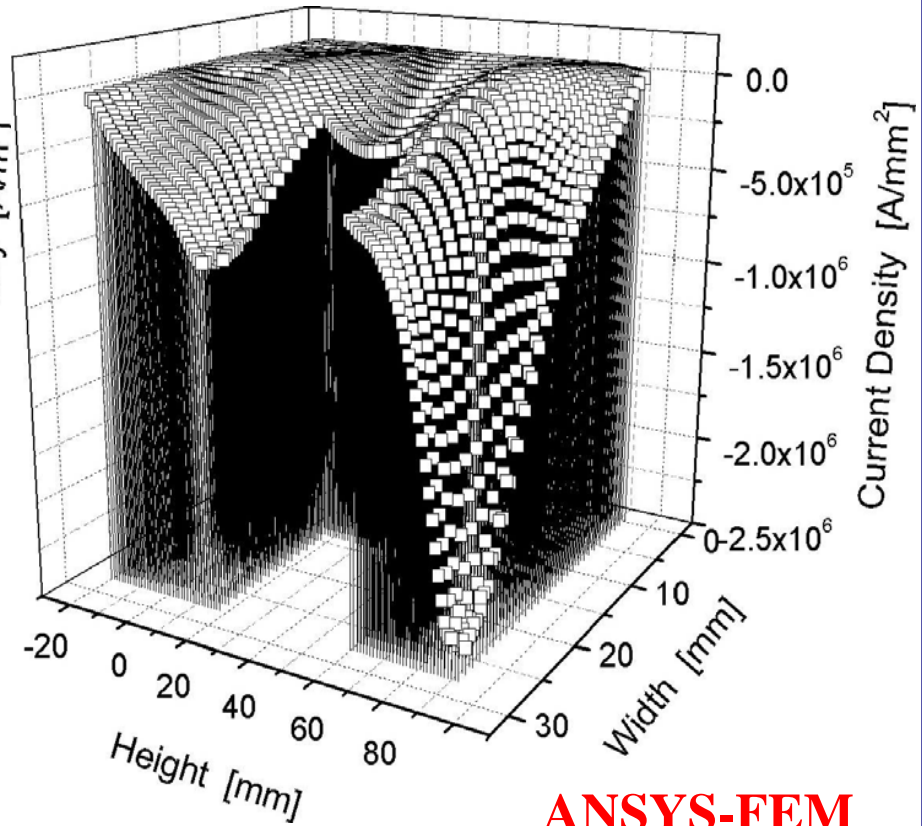
**a)** Meshed grids for ANSYS **FEM**-analysis, **b)** FEM magnetic flux density vectors, **c)** Meshed grids for the **FDM**-STP simulation, **d)** Converted magnetic flux density vectors



The comparison of the data between before and after conversion (**Joule Heat**)



**FDM**



**ANSYS-FEM**

The comparison of the data between before and after conversion (Inducted current density)

*The physical properties used for the present computer modeling of solidification transport phenomena of  $\gamma(\text{TiAl})\text{-Al}\%$ at. pseudo-binary alloy*

- **Thermal conductivity:**  $\lambda_s(T) = \lambda_L(T) = 2.3 \times 10^{-2}$  [W/mm·°C]
- **Solid specific heat:**

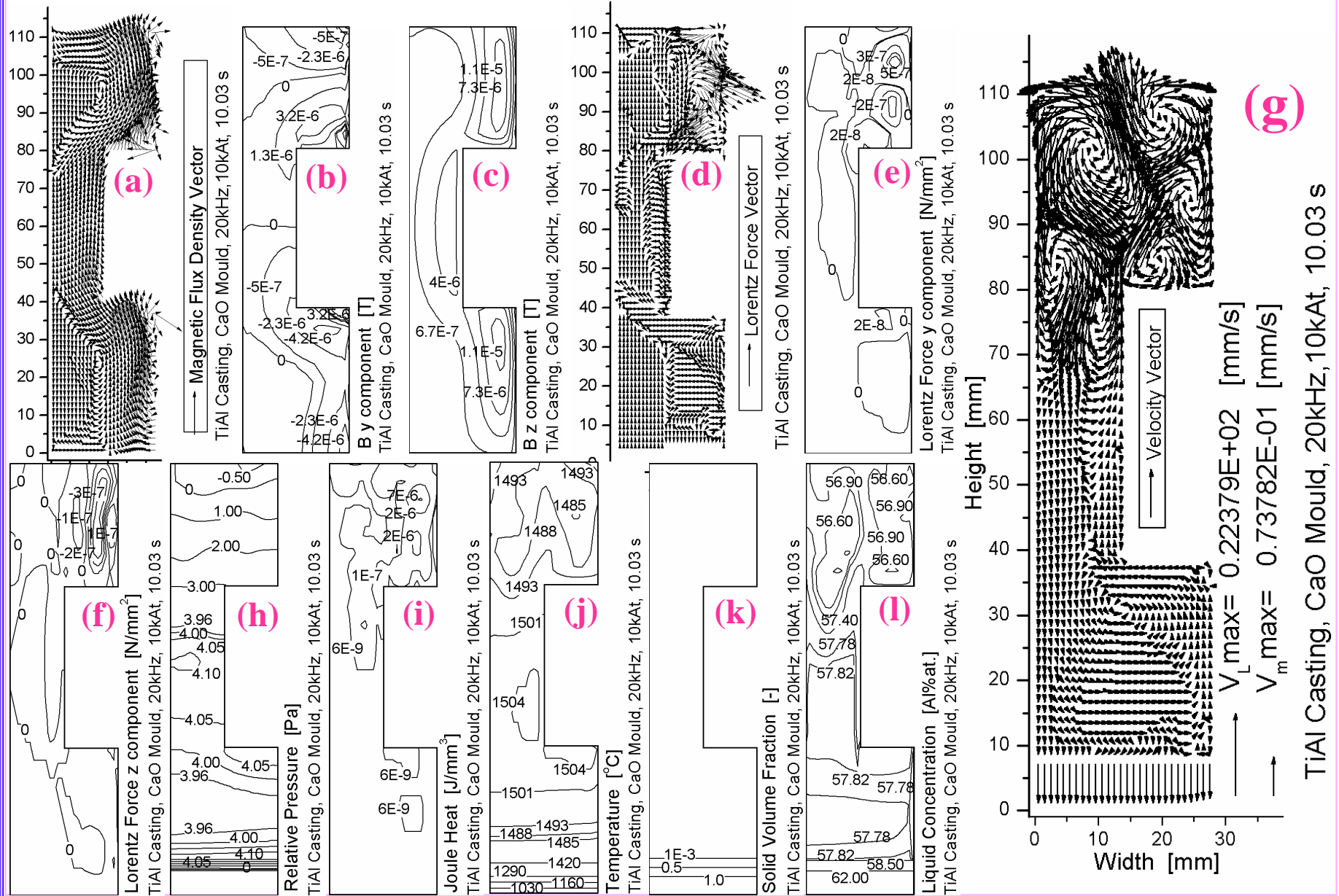
$$c_{PS}(T) = \begin{cases} 0.67832 - 6.4328 \times 10^{-6} \cdot T + 2.5417 \times 10^{-13} \cdot T^3, & (25^\circ\text{C} < T \leq 660^\circ\text{C}) \\ 0.68286 + 1.3644 \times 10^{-5} \cdot T, & (660^\circ\text{C} < T \leq 882^\circ\text{C}) \\ 0.65193 + 1.0513 \times 10^{-5} \cdot T, & (882^\circ\text{C} < T \leq 1490^\circ\text{C}) \end{cases}$$
 [J/g·°C]
- **Liquid specific heat:**  $c_{PL}(T, C_L) = 0.86074$ , (1490°C < T ≤ 2727°C) [J/g·°C]
- **Solid density of the alloy:**  $\rho_s(T, C_L) = 3.8 \times 10^{-3}$ , [g/mm<sup>3</sup>]
- **Liquid density of the alloy:**

$$\rho_L(T, C_L) = 3.632 \times 10^{-3} - 2.0 \times 10^{-7} \cdot (T - T_{liq.}) - 9.32 \times 10^{-6} \cdot (C_L - C_0)$$
 [g/mm<sup>3</sup>]
- **Latent heat of fusion:**  $h(C_S) = 435.4$  [J/g]
- **Liquidus of the  $\gamma(\text{TiAl})\text{-Al}\%$ at. alloys:**

$$T_{liq}(C_{L/Al}) = 802.19745 + 23.01678 \cdot C_L - 0.20279 \cdot C_L^2$$
, ( $C_L \in [54.86, 74.61 \text{at.}\% \text{Al}]$ ) [°C]
- **Partition coefficient:**

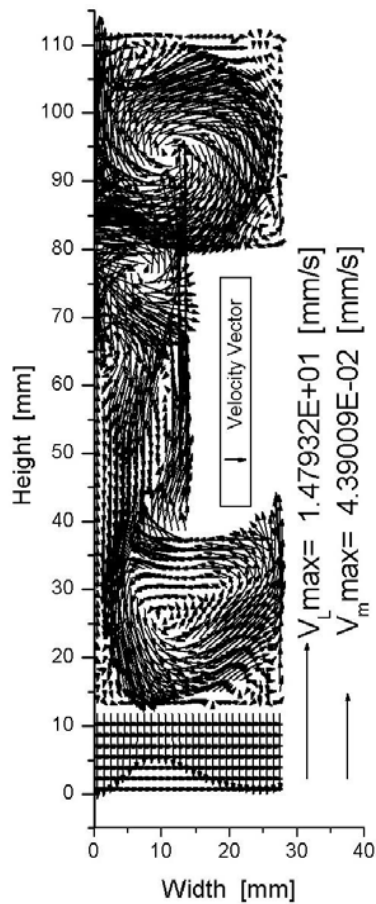
$$k(C_{L/Al}) = 69.24438 - 4.03153 \cdot C_L + 8.894 \times 10^{-2} \cdot C_L^2 - 8.70031 \times 10^{-4} \cdot C_L^3 + 3.18595 \times 10^{-6} \cdot C_L^4$$
  
 $(C_L \in [54.86, 74.61 \text{at.}\% \text{Al}])$  [-]
- **Liquid diffusion coefficient:**  $D_{L/Al}(T^\circ\text{C}) = 5 \times 10^{-3}$  [mm<sup>2</sup>/s]
- **Solid diffusion coefficient:**  $D_{S/Al}(T^\circ\text{C}) = 0.11 \cdot \exp[-14504/(T+273.15)]$  [mm<sup>2</sup>/s]
- **Dynamic viscosity coefficient:**  $\mu(T) = 3.5 \times 10^{-3}$  [Pa·s] (= [N·s/m<sup>2</sup>] = [g/mm·s])
- **Permeability coefficient:**

$$K = \begin{cases} 4.0 \times 10^{-4} f_L^{3.0}, & (f_L \leq 0.7088) \\ 1.6318 f_L^{27.155}, & (0.7088 < f_L \leq 0.99999) \\ 1.0 \times 10^{18}, & (f_L > 0.99999) \end{cases}$$
 [mm<sup>2</sup>]
- **Primary dendrite spacing:**  $d_1 = 1.4 \times 10^{-1}$  [mm]
- **Secondary dendrite arm spacing:**  $d_2 = 3.0 \times 10^{-2}$  [mm]

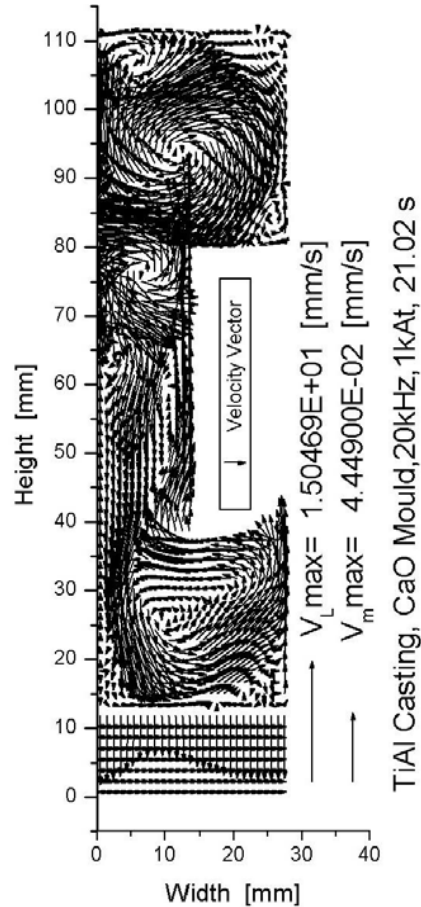


Directional STP of a  $\gamma(\text{TiAl})$ -55at.%Al shaped casting under a harmonic EM-field: (a) FDM-converted/interpolated  $B$  vectors from ANSYS 6.1 FEM-results; (b) y-component of the  $B$  field; (c) z-component of the  $B$  field; (d) Lorentz force vectors; (e) y-component of the Lorentz force; (f) z-component of the Lorentz force; (g) velocity vectors of the alloy melt; (h) relative pressure contours of liquid phase; (i) contours of induced Joule heat; (j) temperature contours; (k) contours of solid volume fraction; (l) liquid concentration contours for solute Al (at.%).

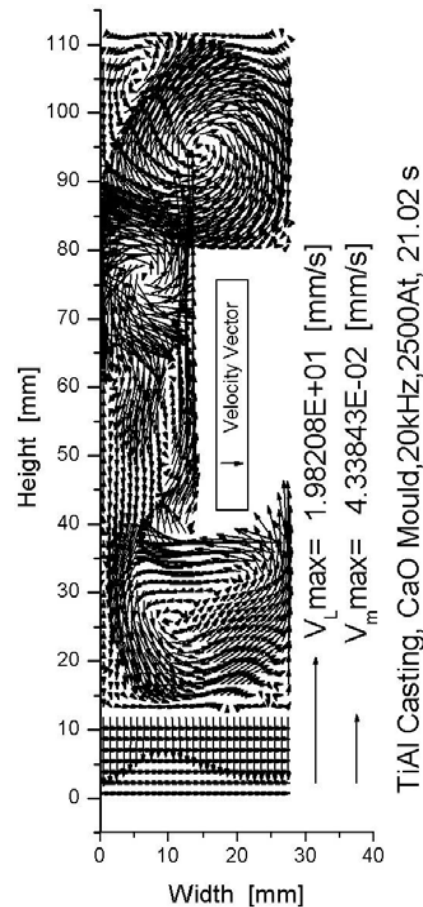
# Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads. $\gamma(\text{TiAl})\text{-}55\text{Al}\%$ at., 20kHz



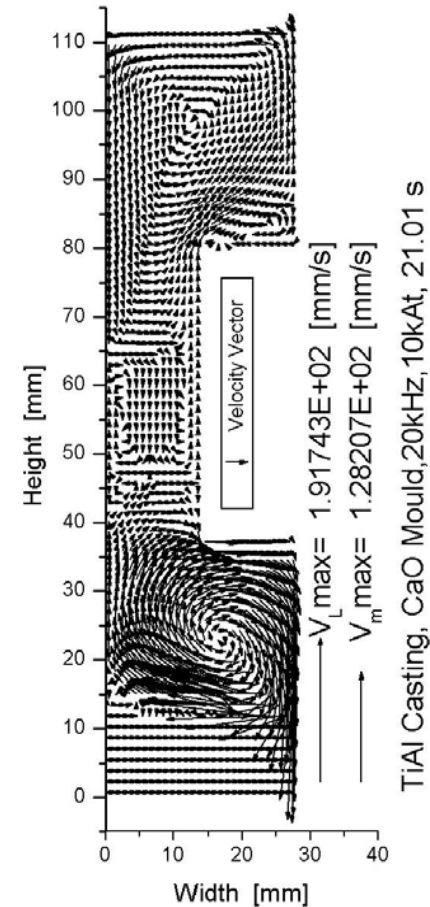
0At



1000At



2500At

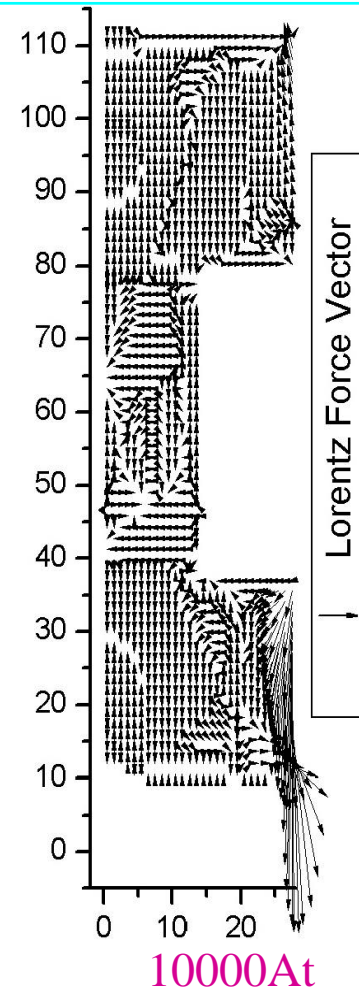
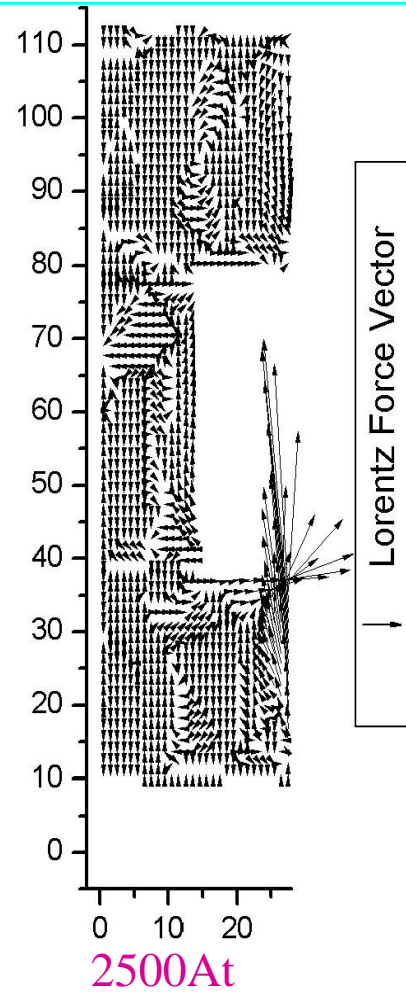
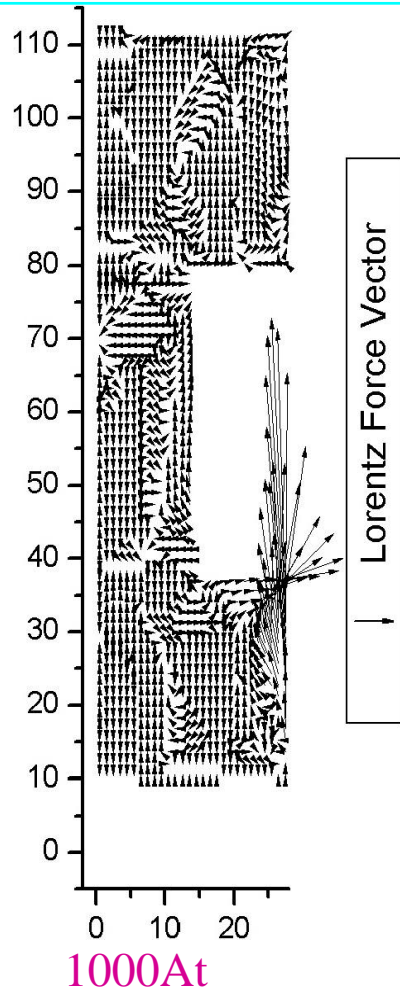


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Melt Flow Velocity Vectors at ~21.0 sec

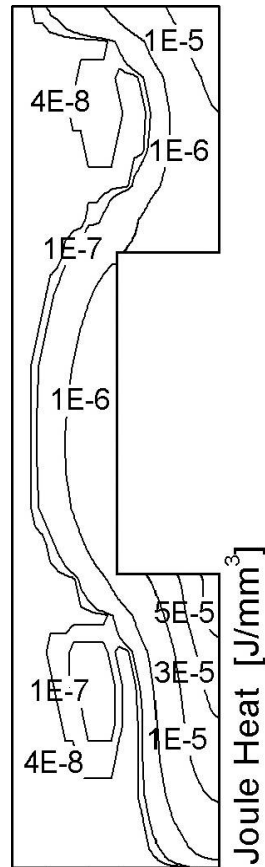


**Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads.  $\gamma(\text{TiAl})\text{-}55\text{Al}\%$ at. , 20kHz**

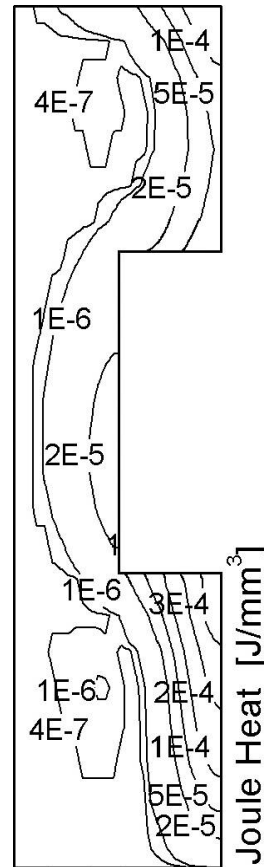


**Lorentz Force Vectors at ~21.0 sec**

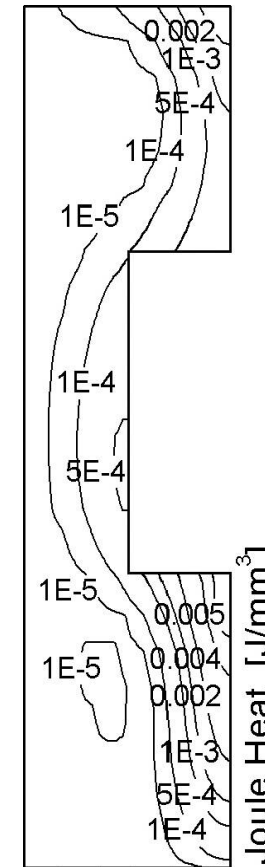
**Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads.  $\gamma(\text{TiAl})\text{-}55\text{Al}\%_{\text{at.}}$ , 20kHz**



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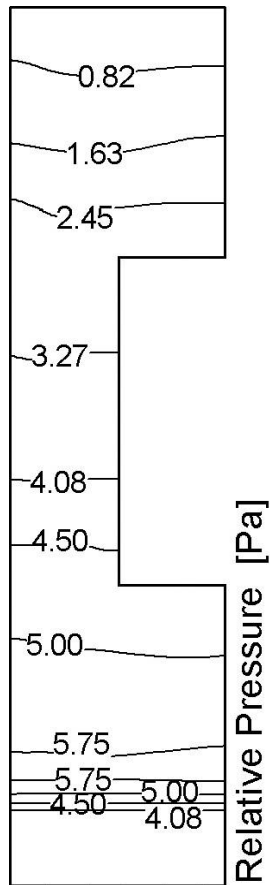
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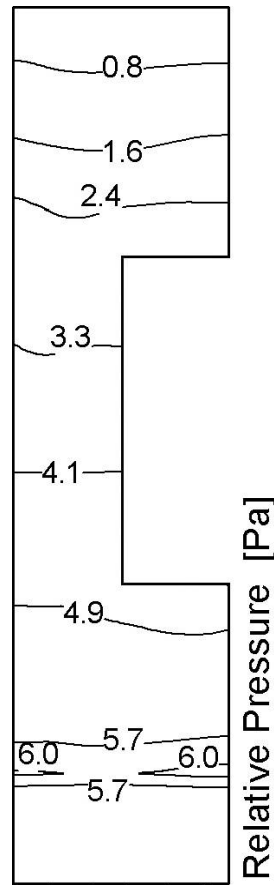
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**Contours of Joule Heat at ~21.0 sec**

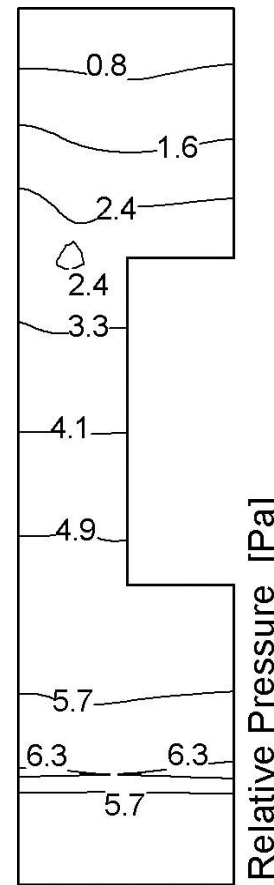
**Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads.  $\gamma(\text{TiAl})\text{-}55\text{Al}\%_{\text{at.}}$ , 20kHz**



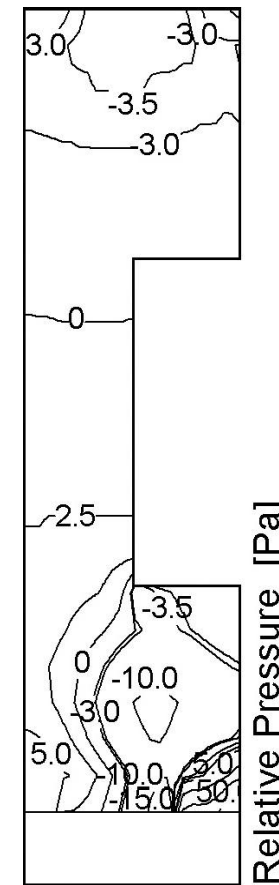
0At



1000At



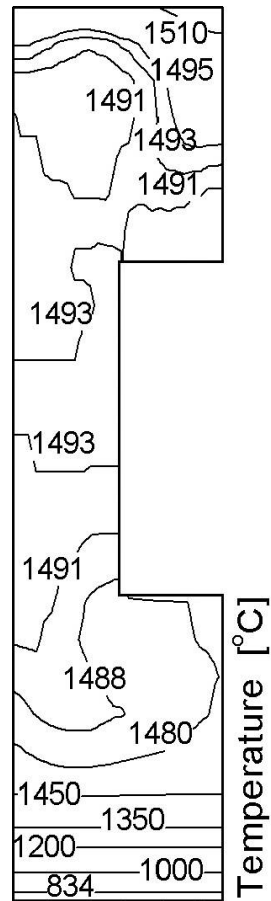
2500At



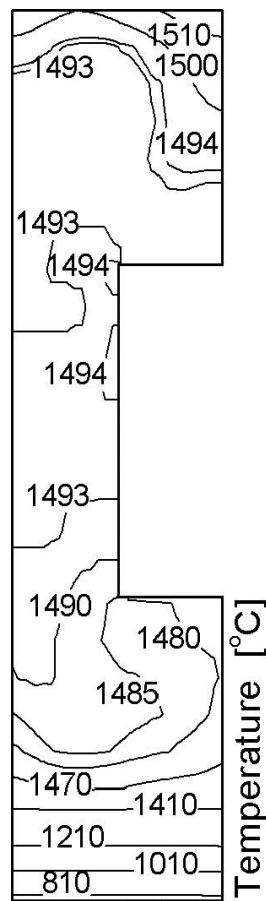
10000At

**Contours of Relative Pressure at ~21.0 sec**

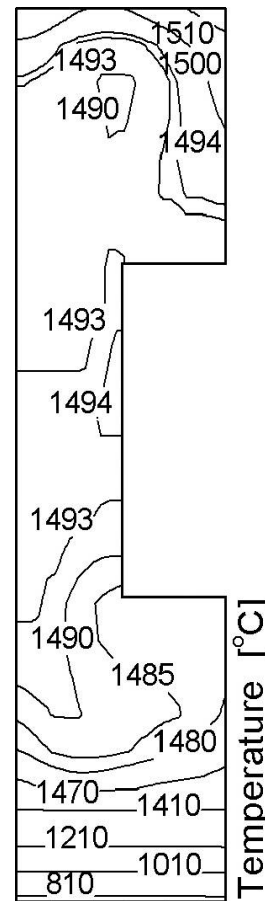
**Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads.  $\gamma(\text{TiAl})\text{-}55\text{Al}\% \text{at.}$ , 20kHz**



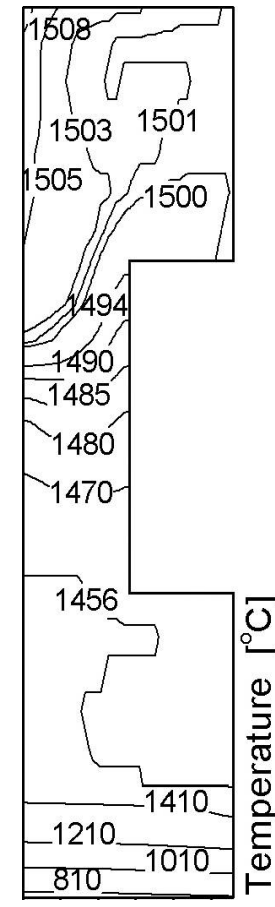
0At



1000At



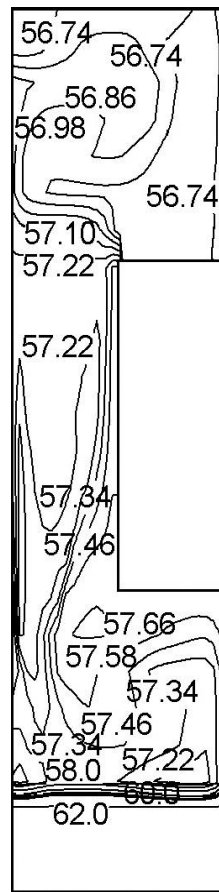
2500At



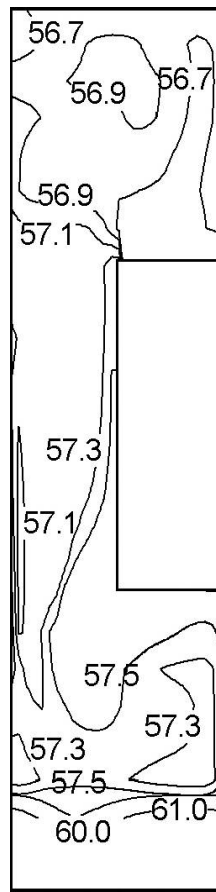
10000At

**Contours of Temperature at ~21.0 sec**

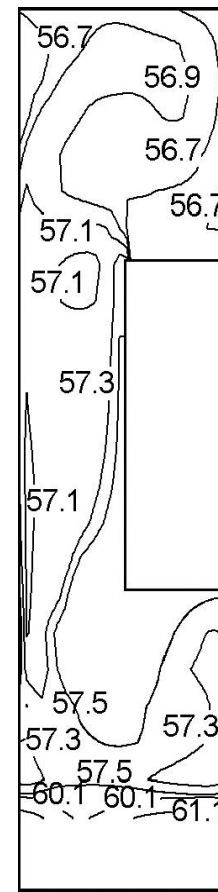
**Directional Solidification Transport Behaviors under Harmonic EM-fields of Different Current Loads.  $\gamma(\text{TiAl})\text{-}55\text{Al}\% \text{at.}$ , 20kHz**



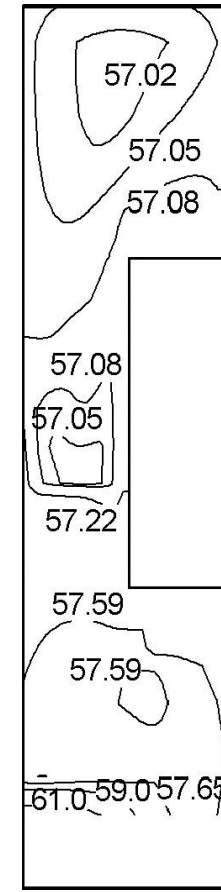
0At



1000At



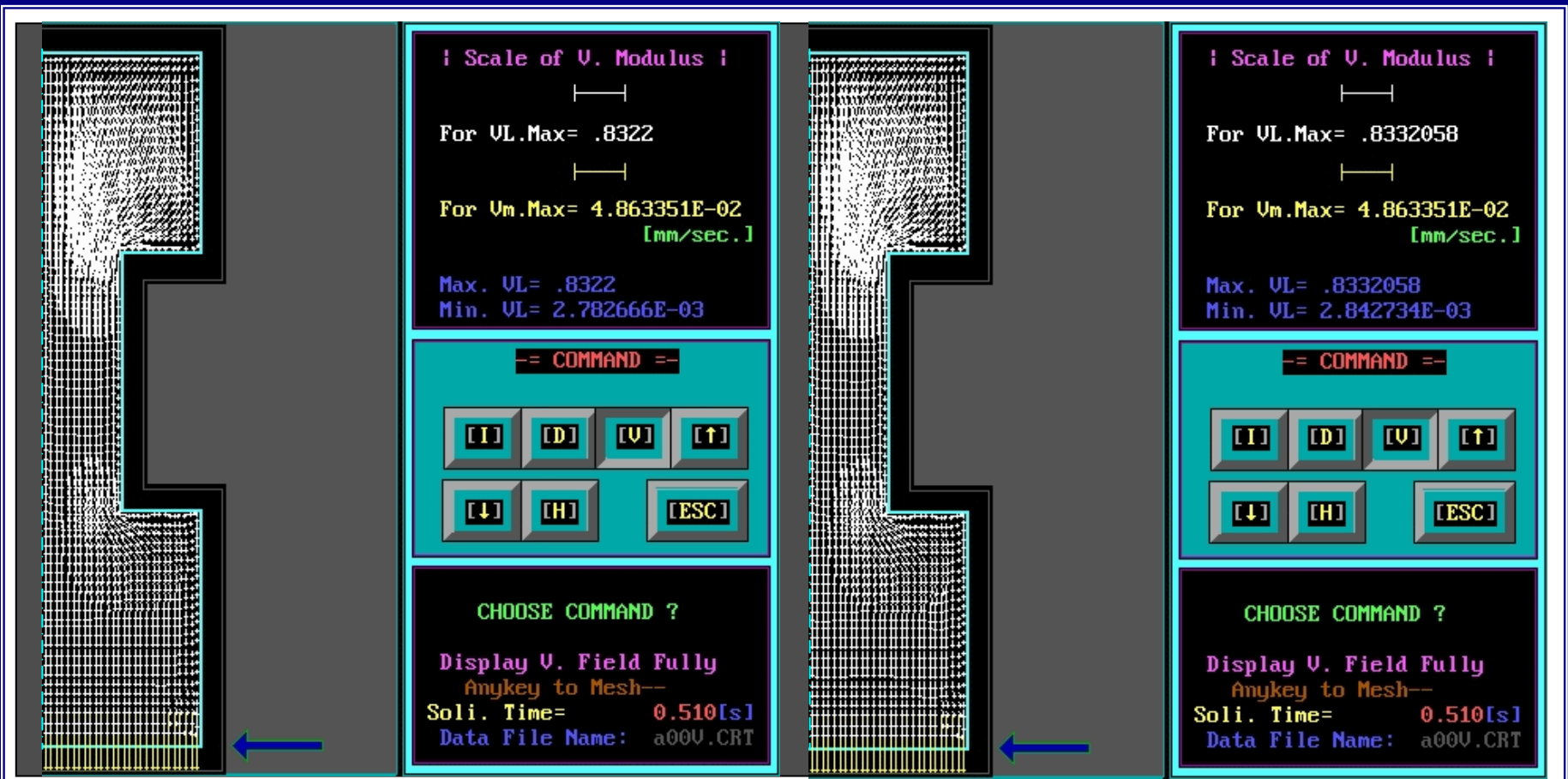
2500At



10000At

**Contours of Liquid Concentration at ~21.0 sec**

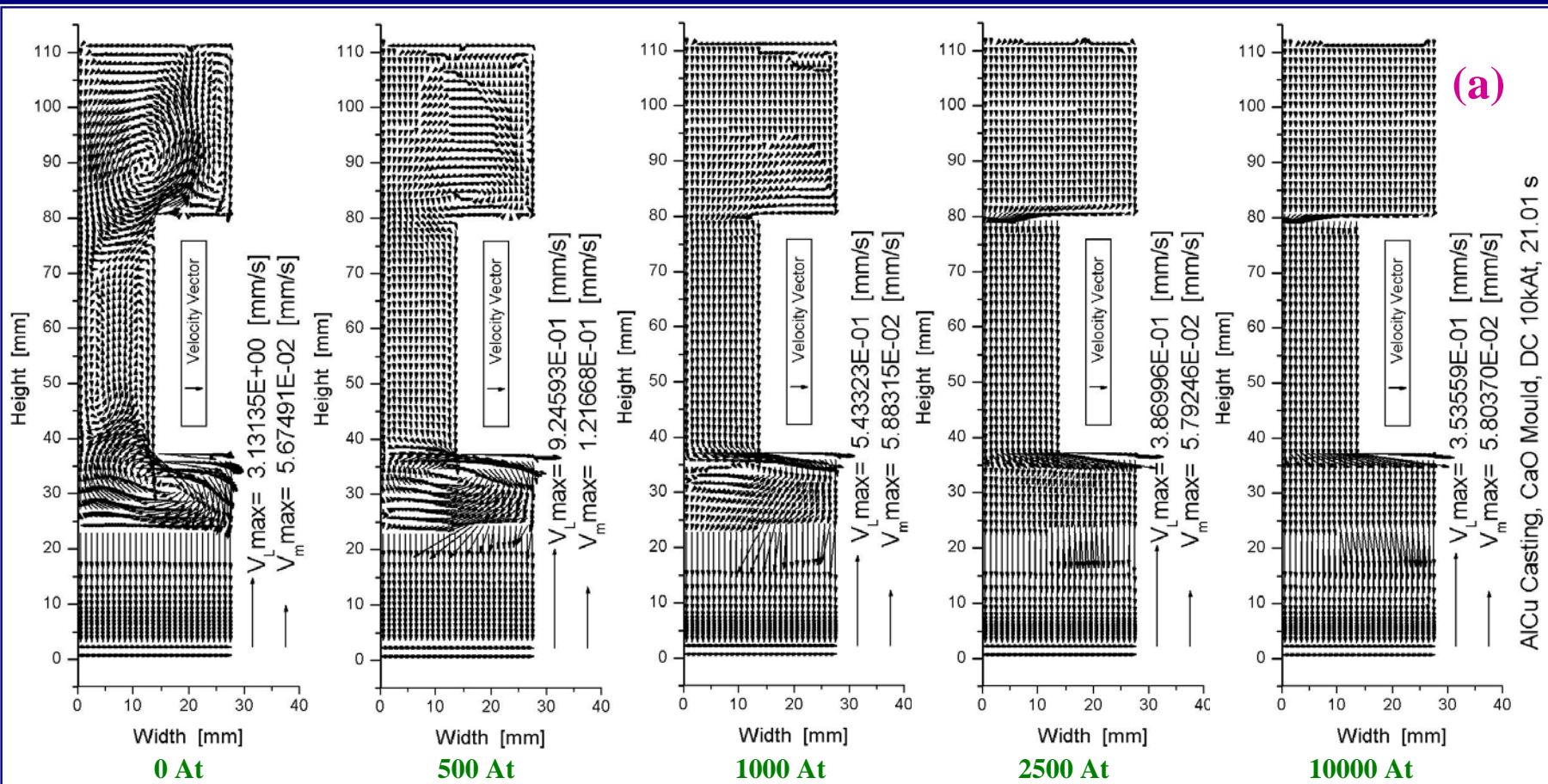
**Animation:**  $V$  &  $f_s$  Evolution in a Blade-like  $\gamma(\text{TiAl})$ -55at.%Al  
Casting directionally Solidifying under Zero & a Harmonic EM-Field



**Only under Gravity**

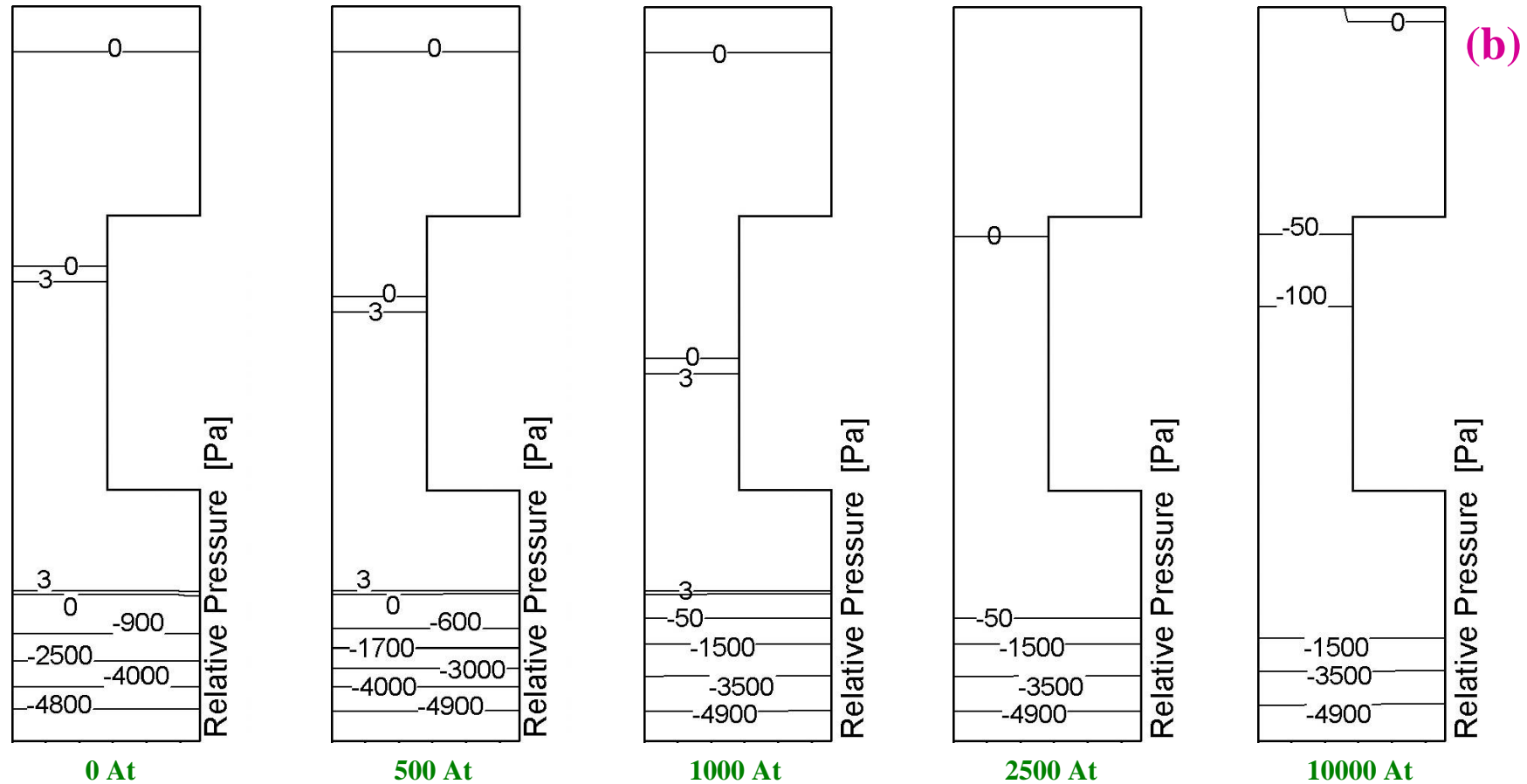
**Under Gravity + Harmonic EM-Field (20kHz, 500kAt)**

## Solidification Transport Processes of a Blade-like Al-4.5%Cu Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)



Comparisons of liquid flow and directional solidification behaviors of Al-4.5%Cu shaped casting under static magnetic fields induced by different current-turns at  $t=21.0$  sec: (a) Velocity vectors of liquid phases; (b) Contours of relative pressure in the liquid; (c) Contours of solid volume fraction; (d) Contours of temperature.

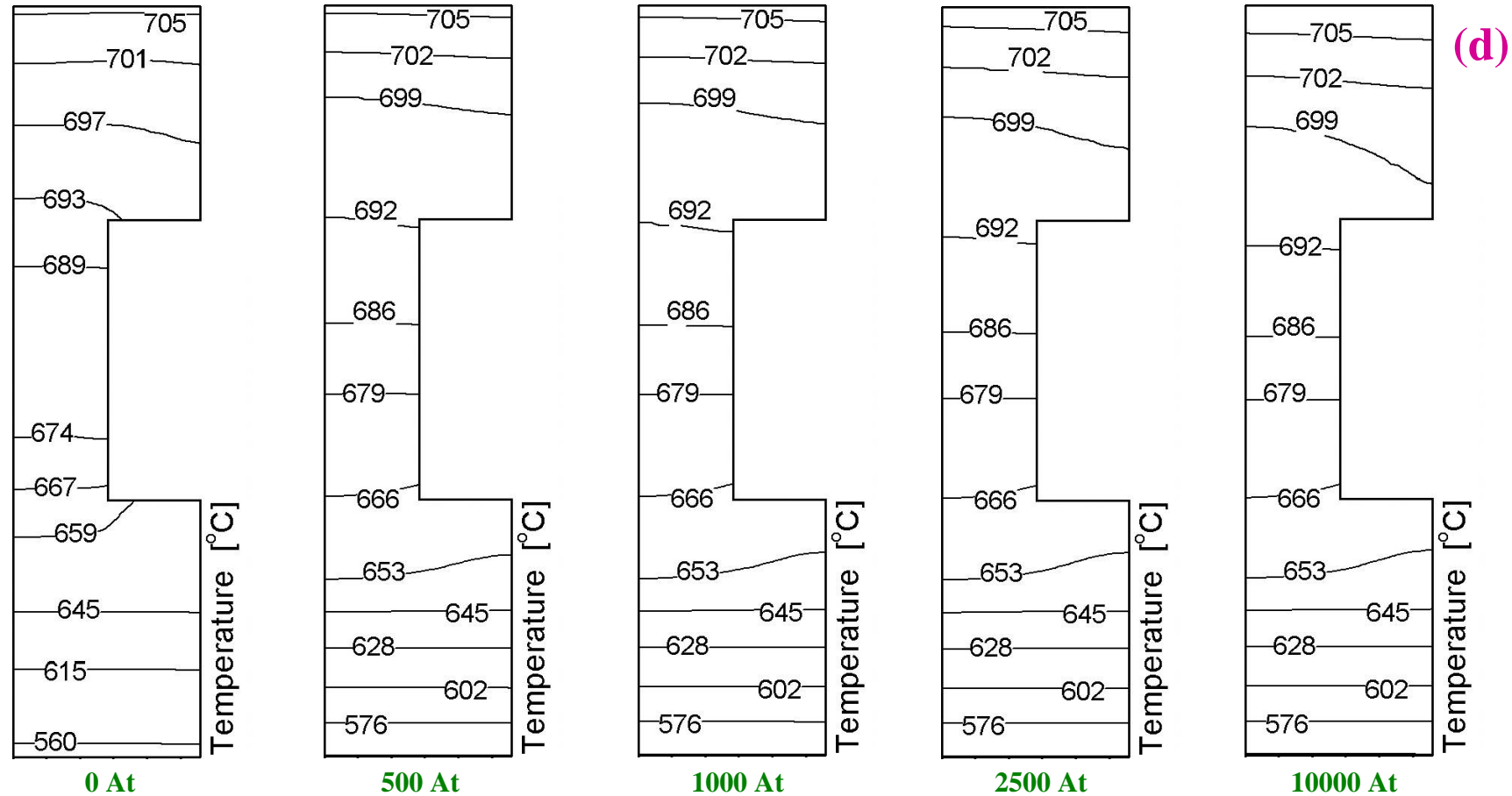
*Solidification Transport Processes of a Blade-like Al-4.5%Cu Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)*



Comparisons of liquid flow and directional solidification behaviors of Al-4.5%Cu shaped casting under static magnetic fields induced by different current-turns at  $t=21.0$  sec: (a) Velocity vectors of liquid phases; (b) Contours of relative pressure in the liquid; (c) Contours of solid volume fraction; (d) Contours of temperature.

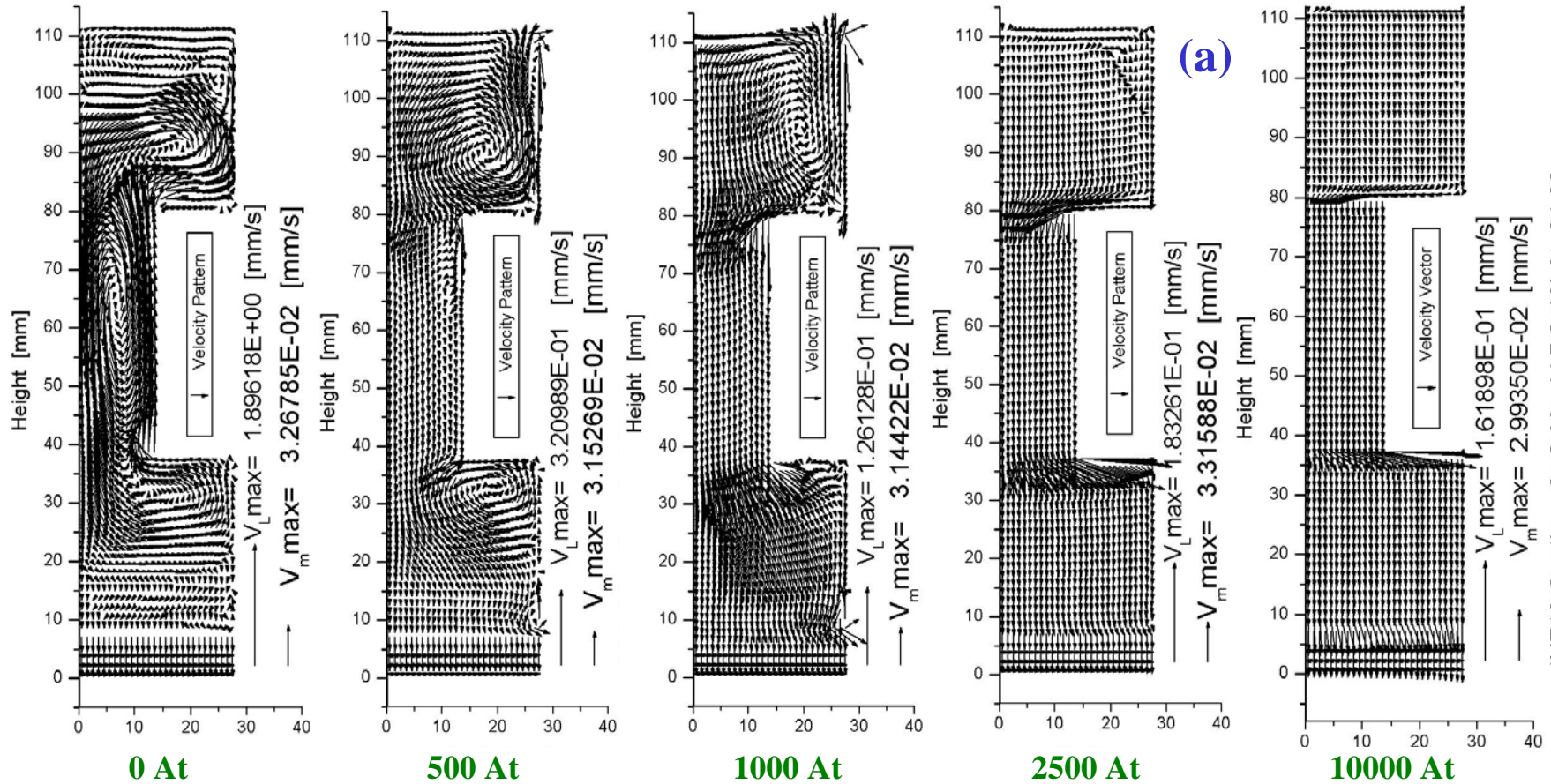


*Solidification Transport Processes of a Blade-like Al-4.5%Cu Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)*



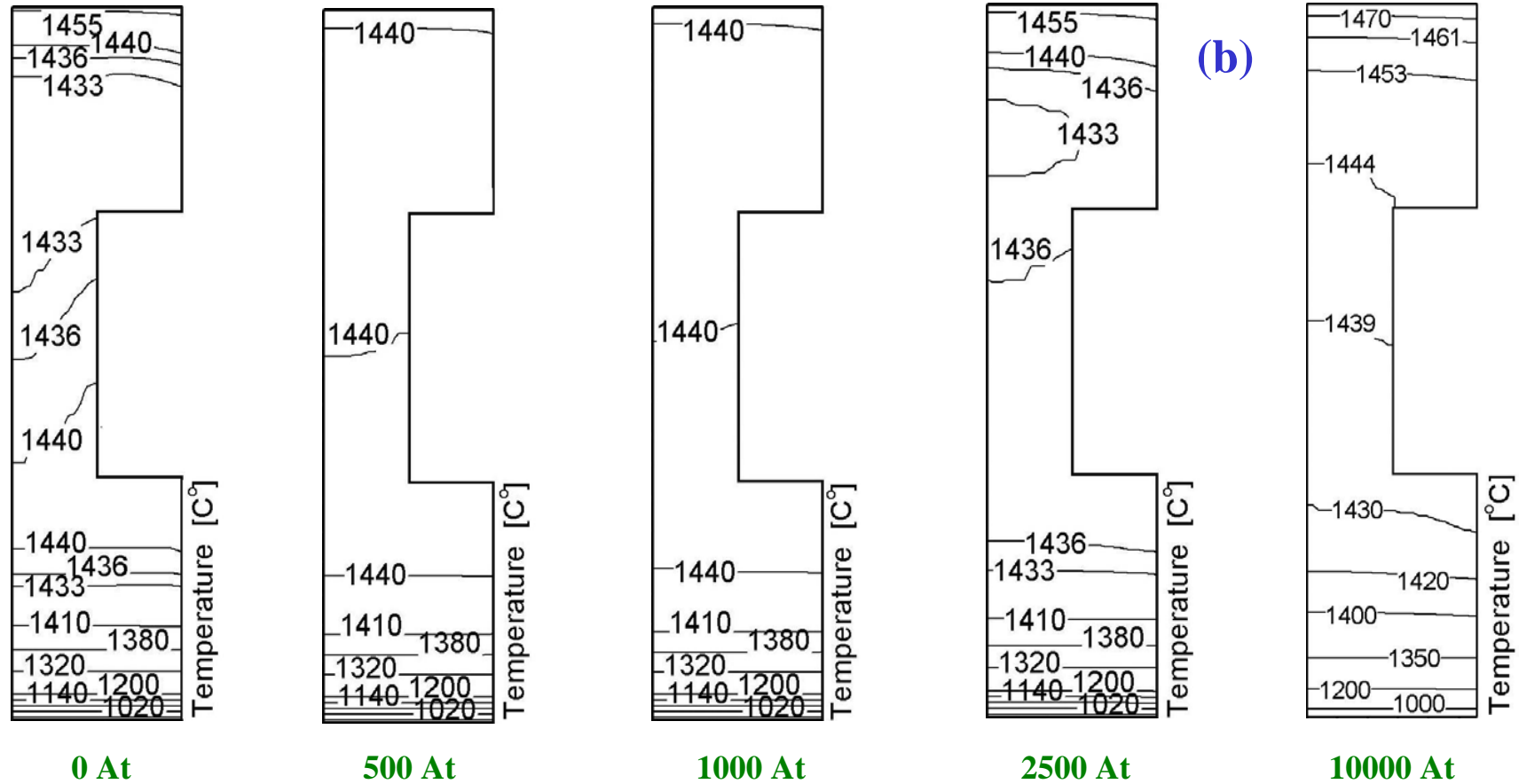
Comparisons of liquid flow and directional solidification behaviors of Al-4.5%Cu shaped casting under static magnetic fields induced by different current-turns at  $t=21.0$  sec: (a) Velocity vectors of liquid phases; (b) Contours of relative pressure in the liquid; (c) Contours of solid volume fraction; (d) Contours of temperature.

**Solidification Transport Processes of a Blade-like In718-4.85%Nb Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)**



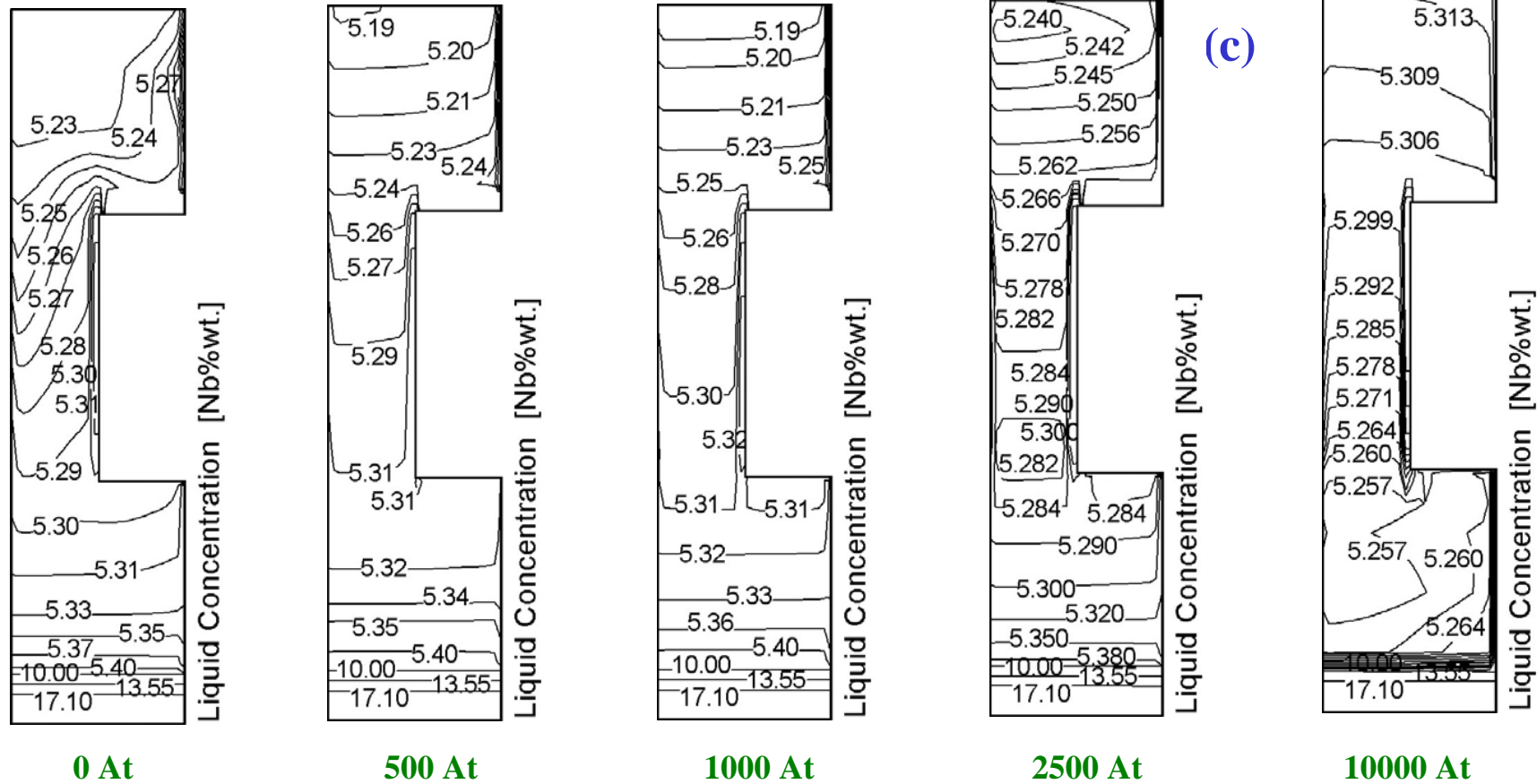
**Influences of transverse static magnetic strengths on the liquid flow behaviors and temperature /concentration distributions in directionally solidified In718 shaped castings at t=21.0 sec.: (a) Velocity vectors of liquid phases; (b) Contours of temperature; (c) Contours of liquid concentration, Nb%wt..**

**Solidification Transport Processes of a Blade-like In718-4.85%Nb Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)**



**Influences of transverse static magnetic strengths on the liquid flow behaviors and temperature /concentration distributions in directionally solidified In718 shaped castings at t=21.0 sec.: (a) Velocity vectors of liquid phases; (b) Contours of temperature; (c) Contours of liquid concentration, Nb%wt..**

**Solidification Transport Processes of a Blade-like In718-4.85%Nb Casting under Transverse Static Magnetic Fields of different Load Strengths (in At)**

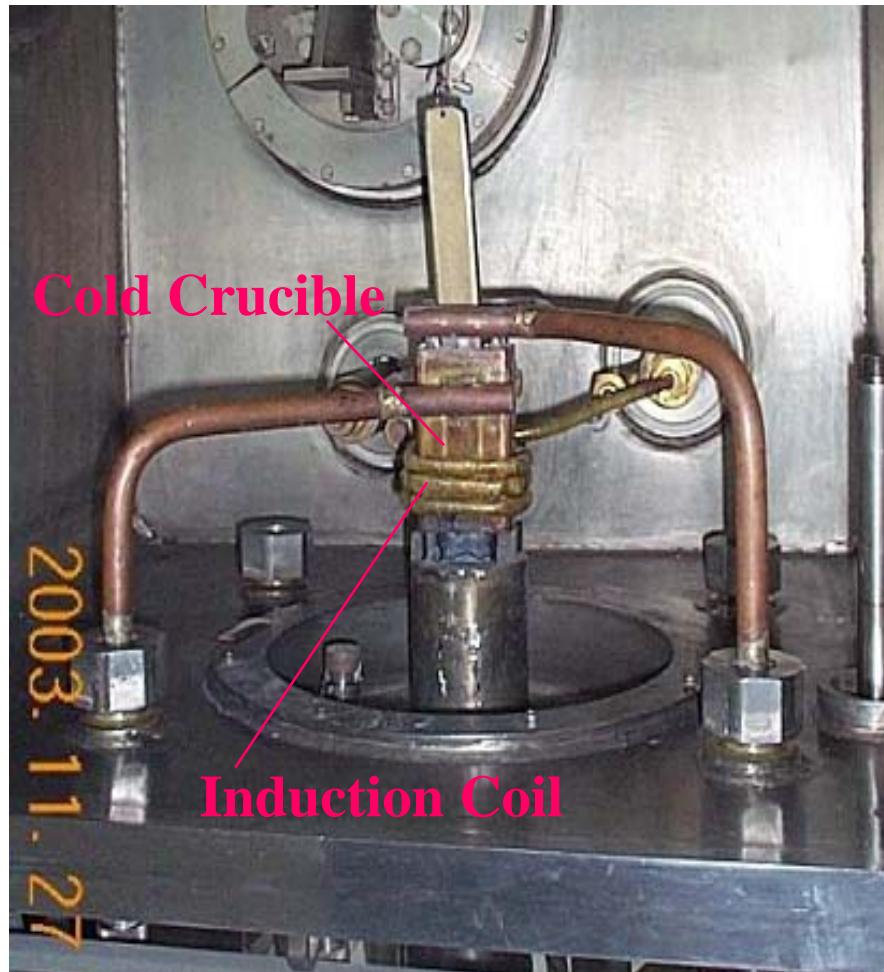


**Influences of transverse static magnetic strengths on the liquid flow behaviors and temperature /concentration distributions in directionally solidified In718 shaped castings at  $t=21.0$  sec.: (a) Velocity vectors of liquid phases; (b) Contours of temperature; (c) Contours of liquid concentration, Nb%wt..**

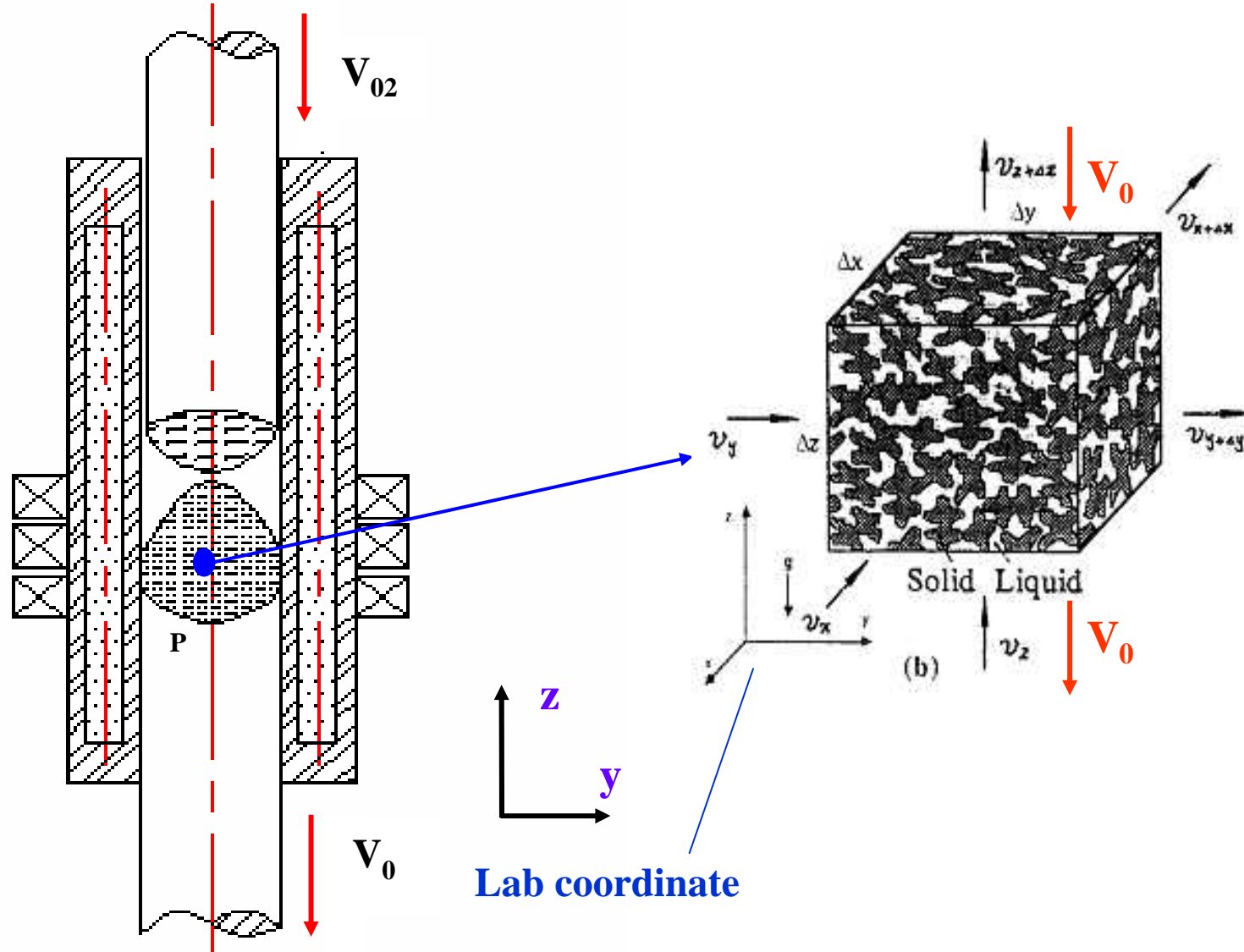
**Application on the Way to:**

*Directional Solidification of Cold  
Crucible Titanium Alloy Strip Ingot*

*Directional Solidification Apparatus of Cold Crucible Ti Alloy Strip Ingot Currently Used in the Authors' Group*







***Schematic EM-Directional Solidification Configuration for Continuum Model Revision***



# Revised Continuum Model for EM-STP

## Solidification heat energy transfer

$$\begin{aligned} & \partial(\rho c_p)_m / \partial t + \nabla(f_L \rho_L c_{pL} VT) + V_0 \partial(f_L \rho_L c_{pL} T) + (f_S \rho_S c_{pS} T) / \partial z \\ & = \nabla[k_m \nabla T] + \rho_S h (\partial f_S / \partial t) + q_J \end{aligned}$$

## Solidification solute mass transfer

$$\begin{aligned} & \partial(\rho C)_m / \partial t + \nabla(f_L \rho_L C_L V) + V_0 \partial(f_L \rho_L C_L) + (f_S \rho_S C_S) / \partial z \\ & = \nabla[\rho_L D_L \nabla(f_L C_L) + \rho_S D_S \nabla(f_S \rho_S C_S)] \end{aligned}$$

## Solidification mass conservation

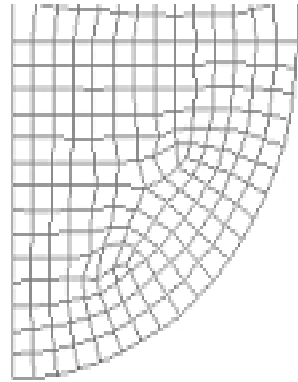
$$\partial \rho / \partial t + \nabla(f_L \rho_L V) + V_0 \partial \rho / \partial z = 0$$

## Momentum transfer for bulk/interdendritic liquid flow

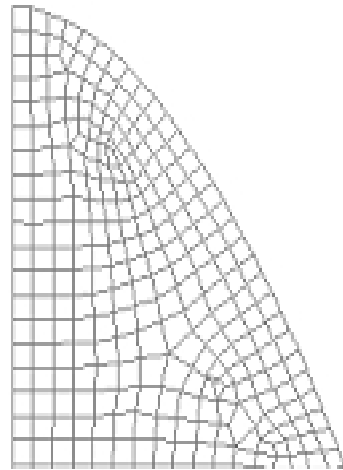
$$\begin{aligned} & \partial f_L \rho_L V / \partial t + \nabla[(f_L \rho_L V) V] + V_0 \nabla(f_L \rho_L V) + V_0 \partial f_L \rho_L V / \partial z \\ & = \nabla[\mu \nabla(f_L V) - \nabla(f_L P) + f_L \rho_L g - (\mu f_L^2 / K) V + f_L \rho_L g + F_L] \end{aligned}$$

# Schematic *FEM* and *FDM* Meshing for the Top and Bottom Hump Regions

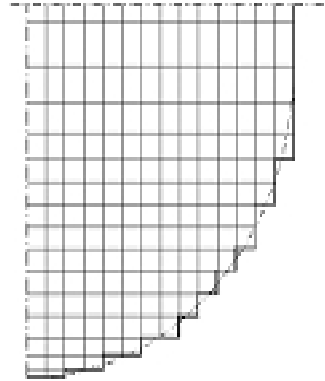
*F*  
*E*  
*M*



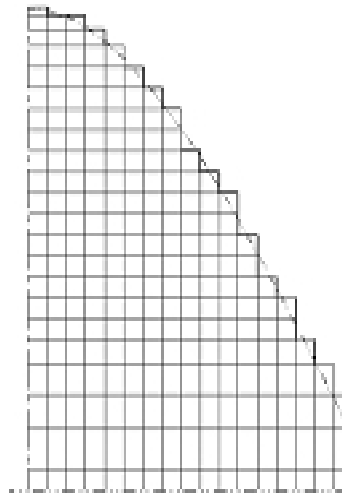
*M*  
*E*  
*S*  
*H*  
*I*  
*N*  
*G*



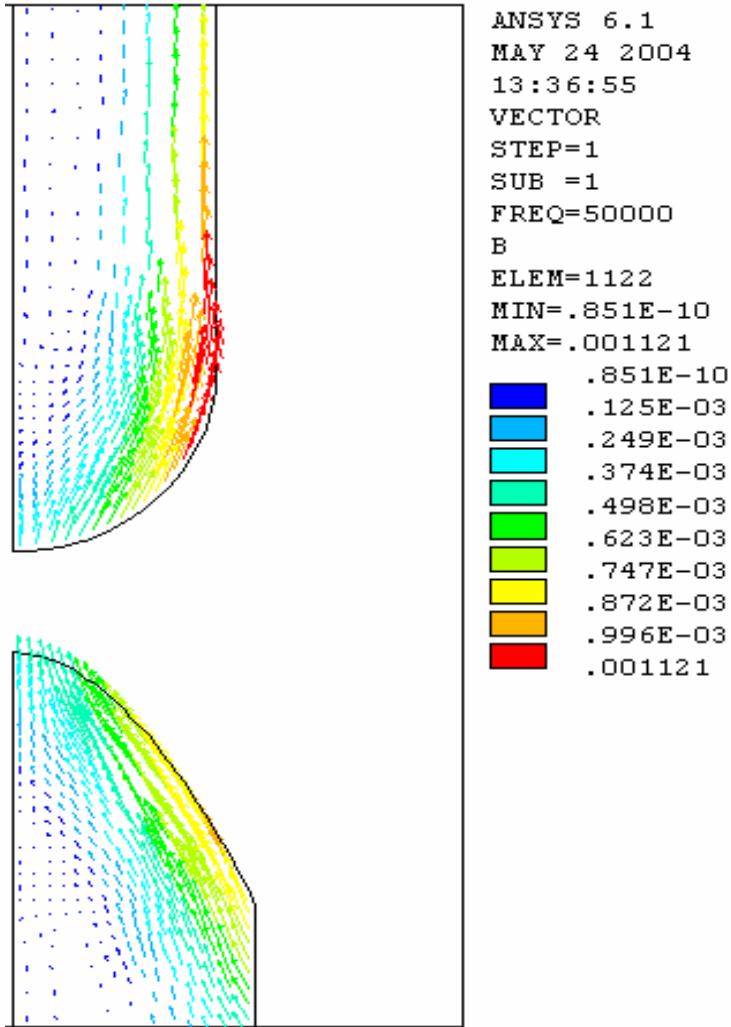
*F*  
*D*  
*M*



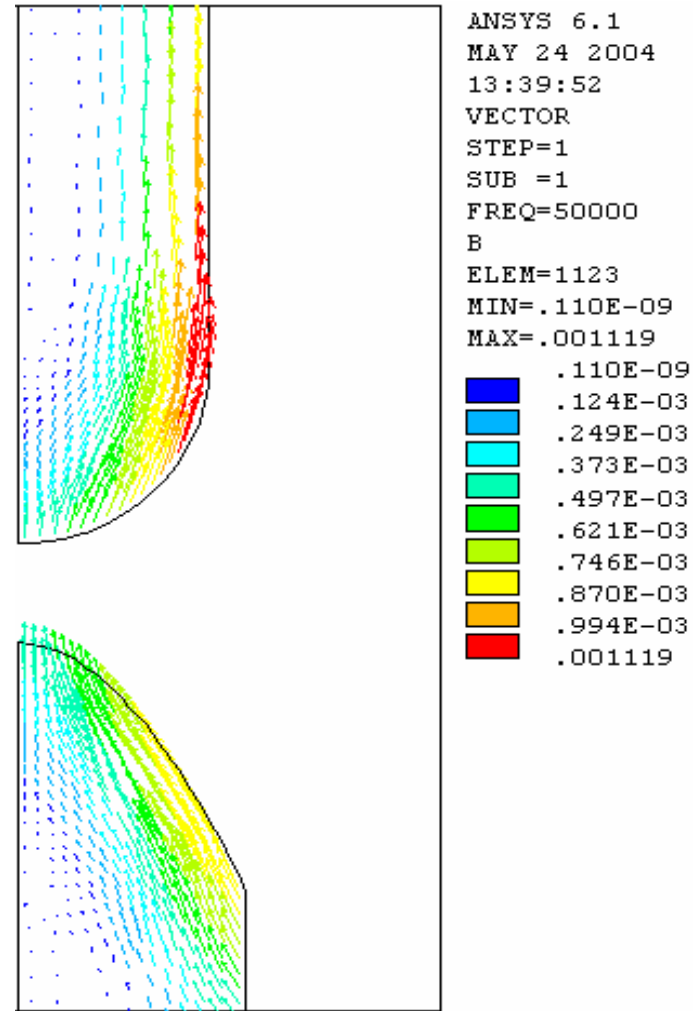
*M*  
*E*  
*S*  
*H*  
*I*  
*N*  
*G*



# Magnetic Flux Density Vectors Output from Ansys 6.1 FEM-Analysis

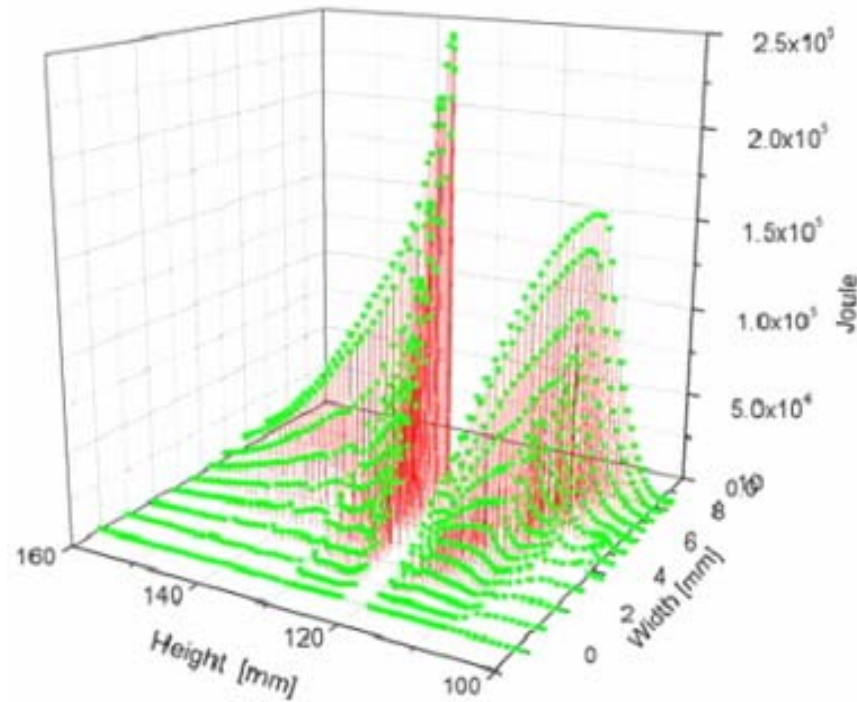


$\rho_E = 2 \times 10^{-5} \Omega m$

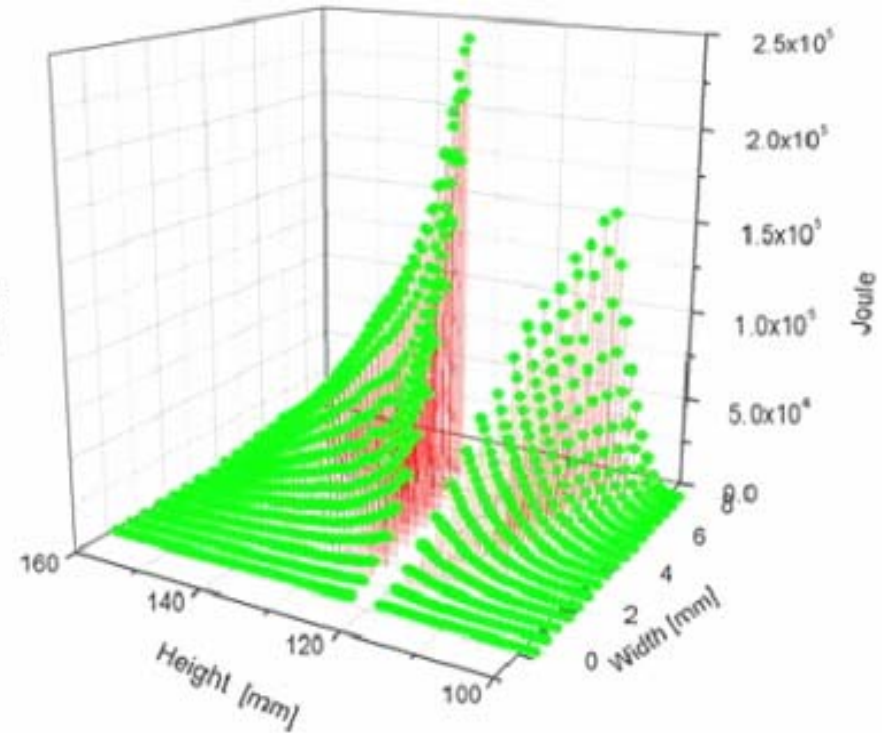


$\rho_E = 2.5 \times 10^{-5} \Omega m$

# *EM-FEM $q_J$ Results $\rightarrow$ FDM Format Conversion*

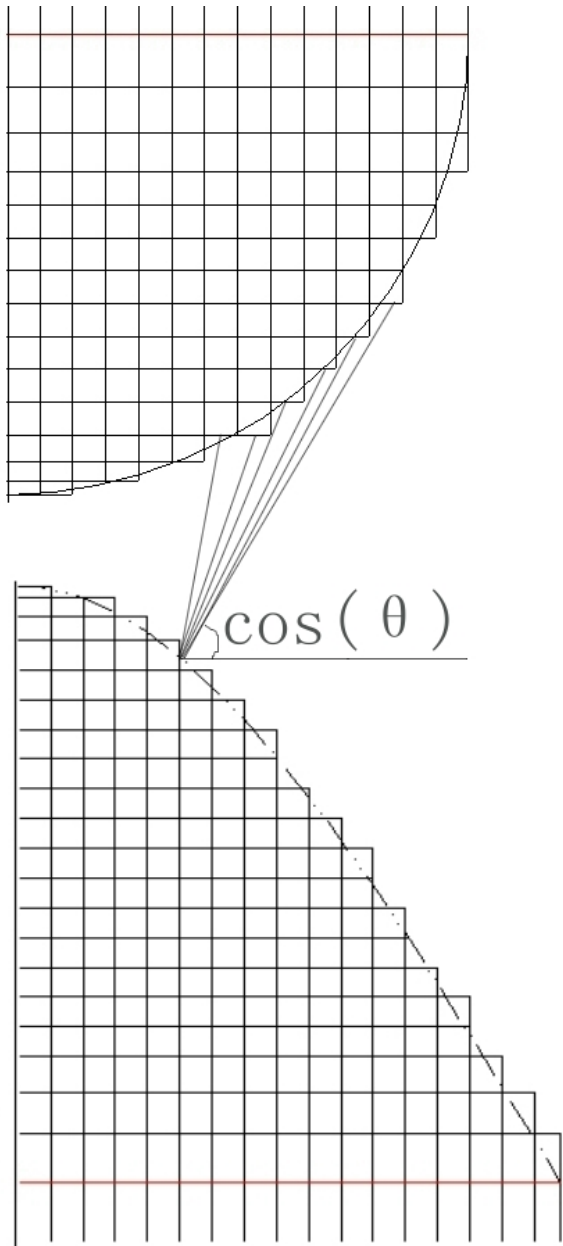


Ansys **FEM**-Results



Converted **FDM** Results

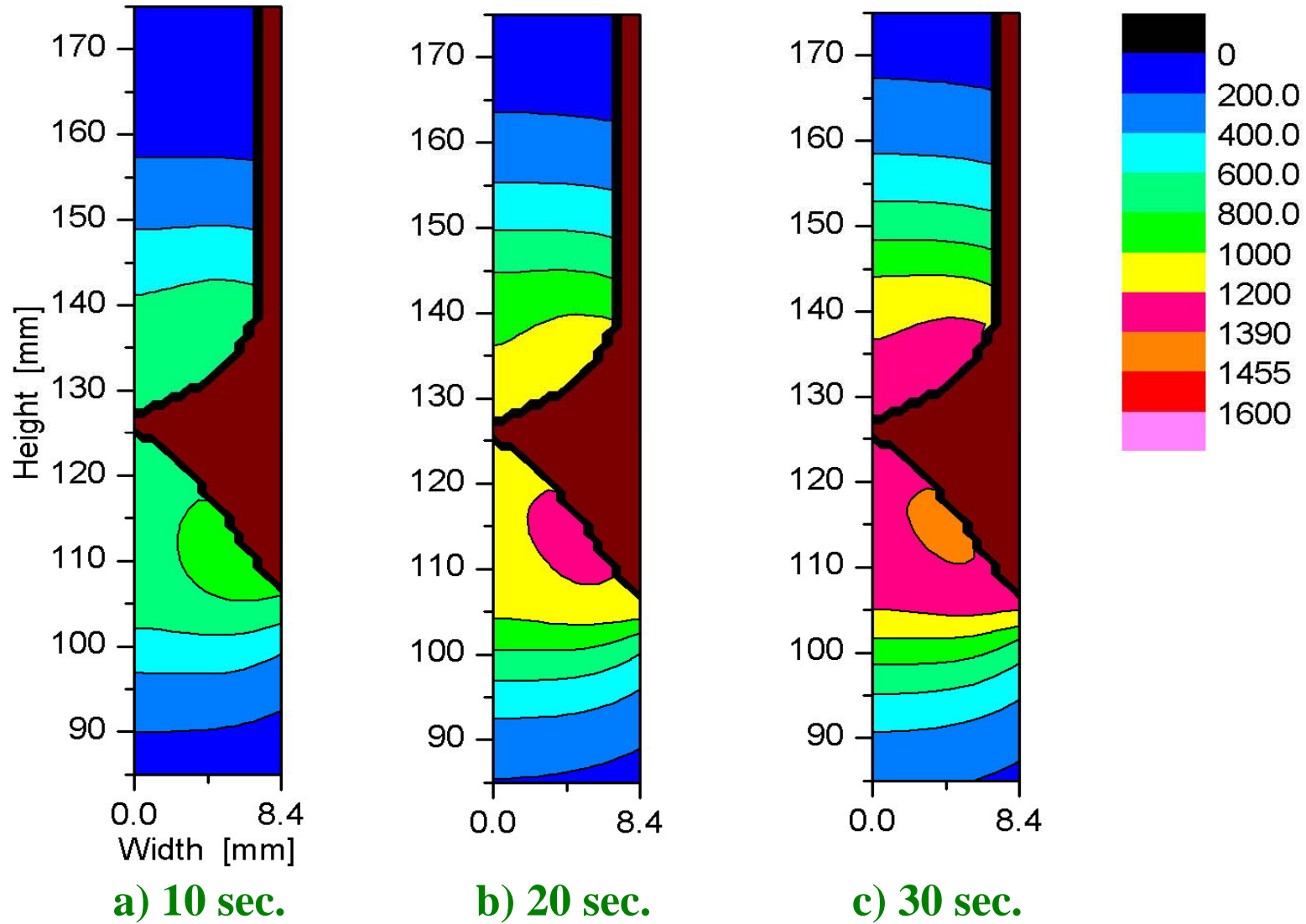
## *Schematic Radiation Heat Transfer between the Top and Bottom Humps*



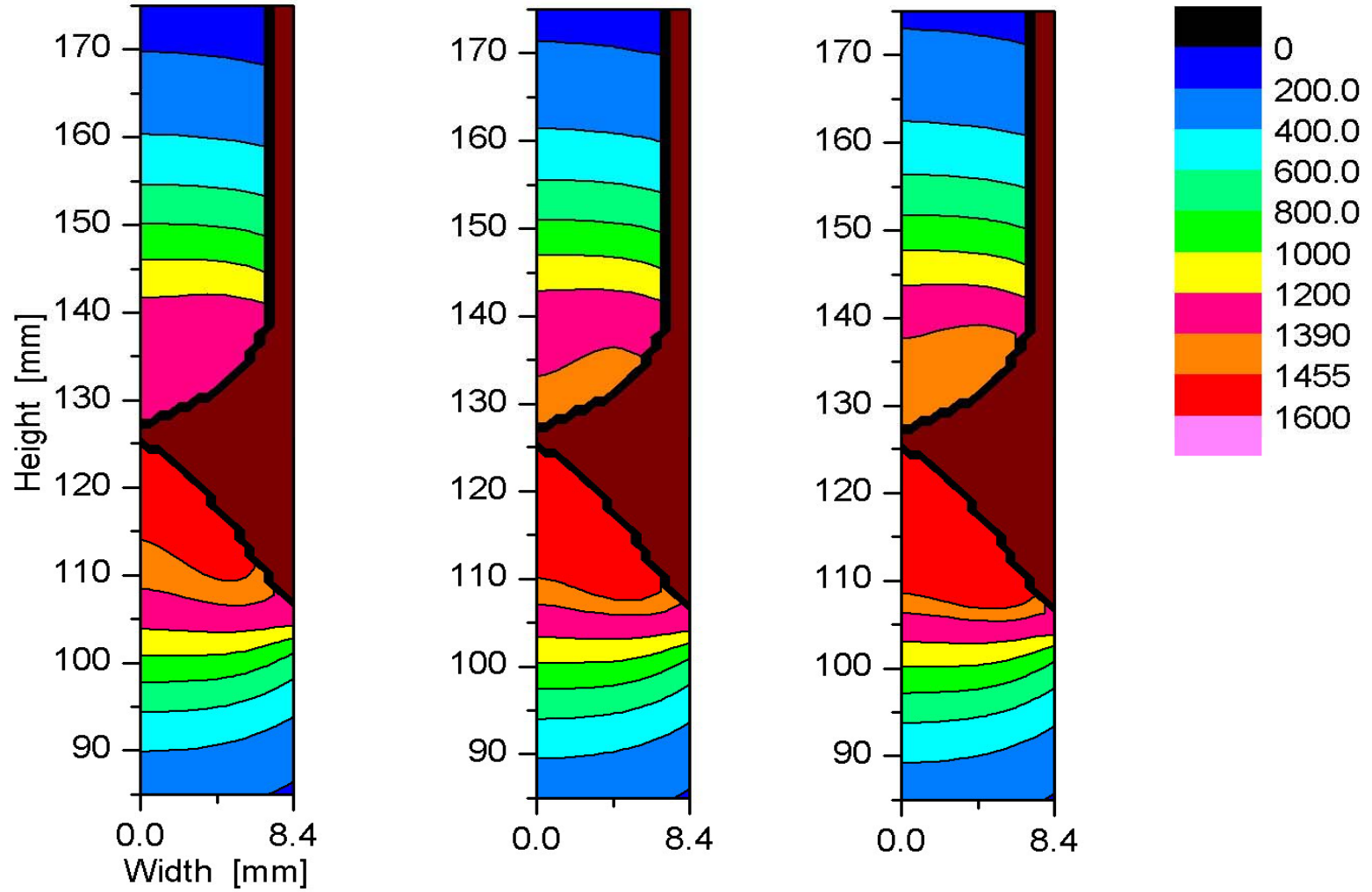
$$T = (\alpha \sum_{J=1}^{J_2} T_{J,K+1/2} + T_M) / [\alpha(J_2 - 1 - J_1) + 1]$$

$\alpha$  is an approximate average of the sum of view factor  $\cos\theta$  ( $0 < \alpha < 1$ )

# EM-Inducted Heating Processes in Top and Bottom Ingots (at $x=12\text{mm}$ )



### EM-Inducted Heating Processes in Top and Bottom Ingots (at $x=12\text{mm}$ )

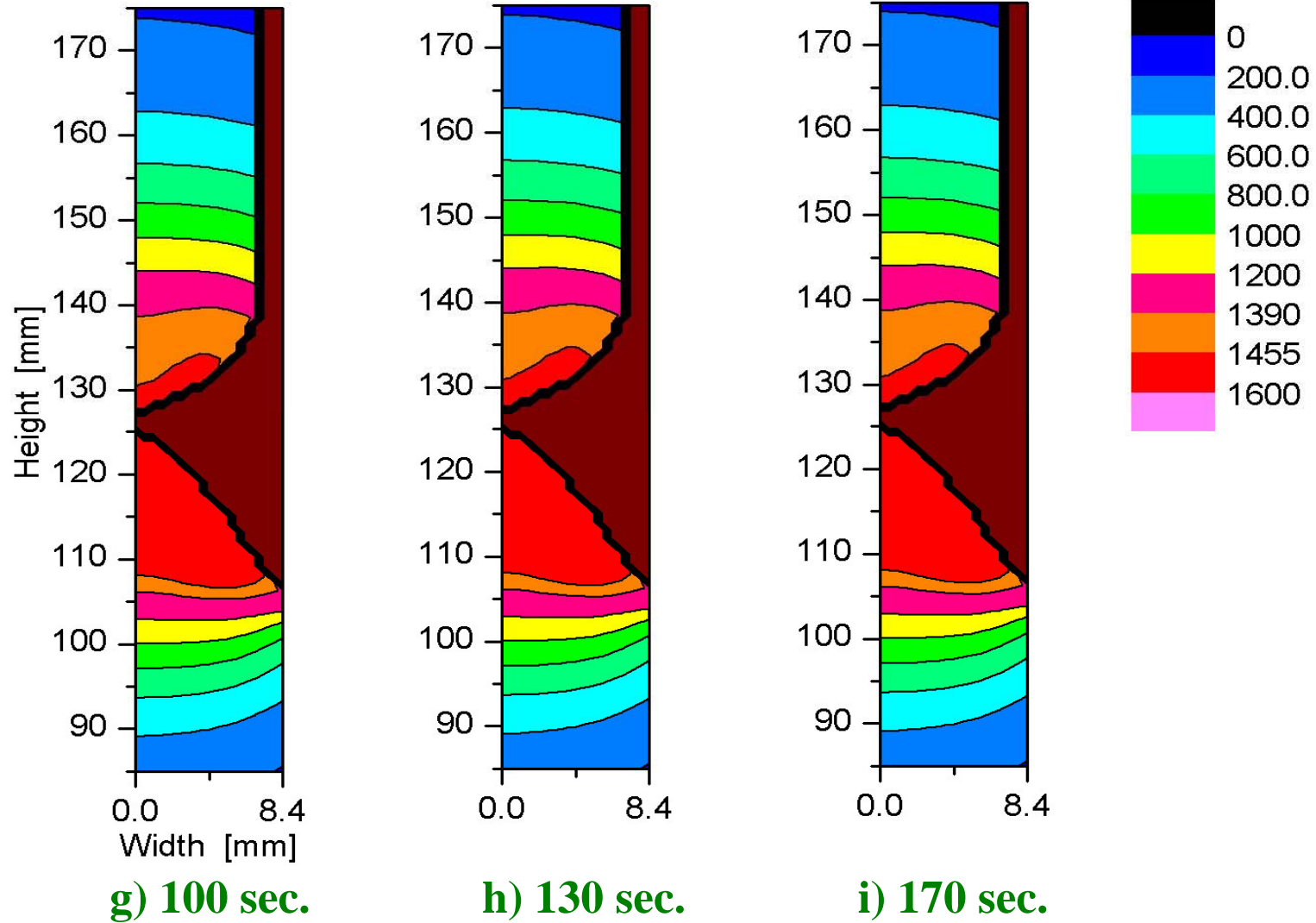


**d) 40 sec.**

**e) 50 sec.**

**f) 60 sec.**

# EM-Inducted Heating Processes in Top and Bottom Ingots (at $x=12\text{mm}$ )





## Summary

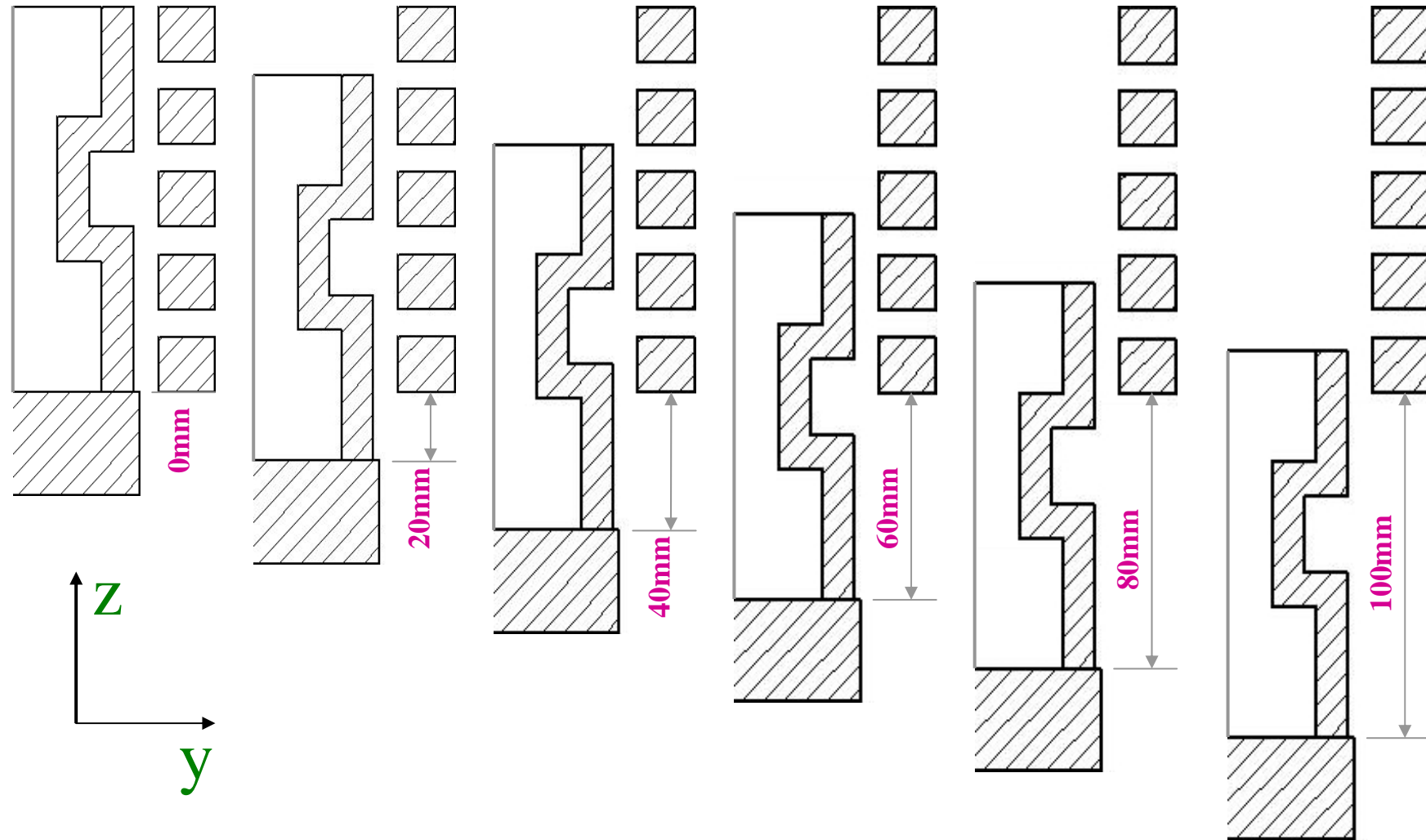
- The proposed modeling methods appear practical and efficient in numerical computation (The each above demonstrated sample usually takes 2-5 hours on a PC with a 1.6GHz-CPU)
- The computational results show that the solidification-shrinkage-driven interdendritic liquid flow in mushy zone, though usually smaller than the convection in bulk liquid region by order(s), can not be easily suppressed by a static magnetic field applied. (The inner force caused by solidification-shrinkage might be much stronger than Lorentz force)

Thank you



*Table I Initial and technological parameters for the present example simulations with 3 different kinds of Alloys*

Alloys	IN718 base-4.85wt.%Nb	$\gamma$ (TiAl)-55at.%Al	Al-4.5wt.%Cu
Pouring temperature, $T_P$	1450 °C	1500 °C	700 °C
Initial mold temperature, $T_M$	1500 °C	1550 °C	700 °C
Heating zone temperature, $T_h$	1600 °C	1600 °C	950 °C
Cooling zone temperature, $T_C$	45 °C	45 °C	25 °C
Bottom cooler temperature, $T_{BC}$	45 °C	45 °C	25 °C
Withdrawal velocity, $V_0$	0.15 mm/sec	0.15 mm/sec	0.15 mm/sec



*Configurations of Directionally Solidifying Blade-like Alloy Casting under a Harmonic EM-Field at Different Withdrawal Positions —At Each Position, an Ansys EM-FEM Analysis Made for EM-Field Interpolations for the Continuous EM-Solidification Process*

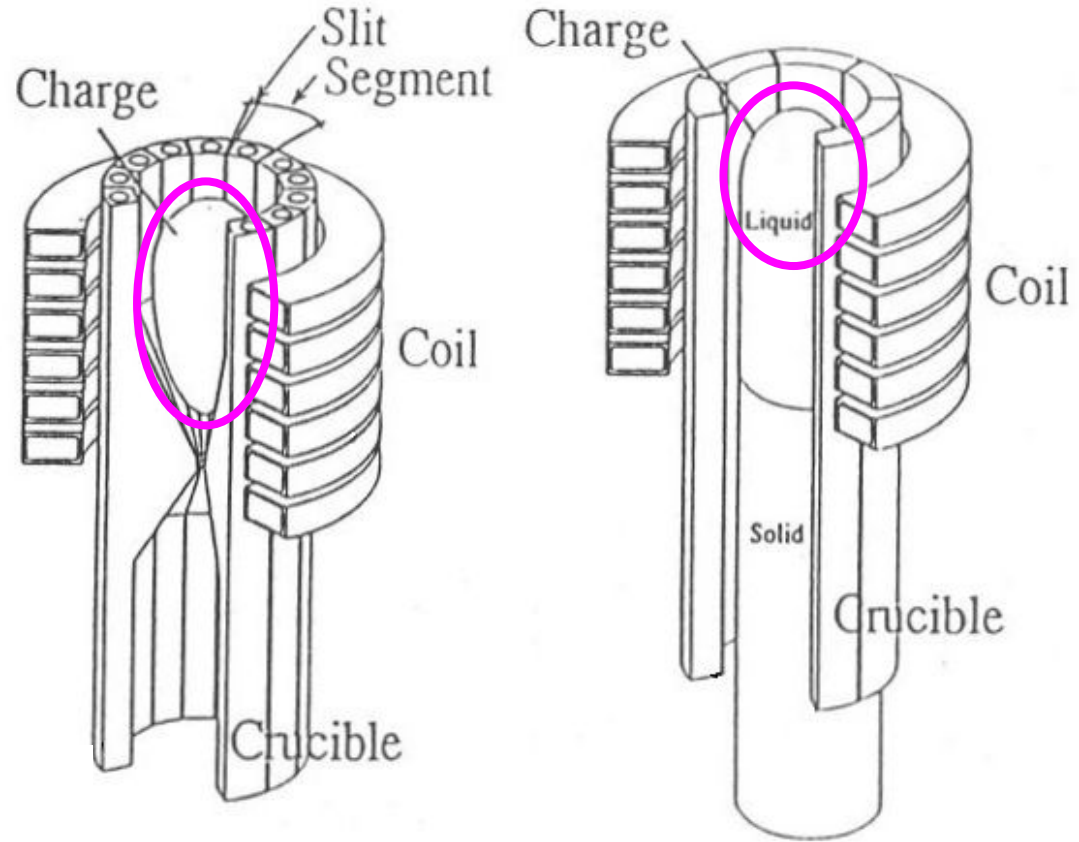
# Free Surface Tension:

Determining the free surface morphology of alloy melt corporately with gravity/ Lorentz force and the dynamic pressure resulted from the melt flow.

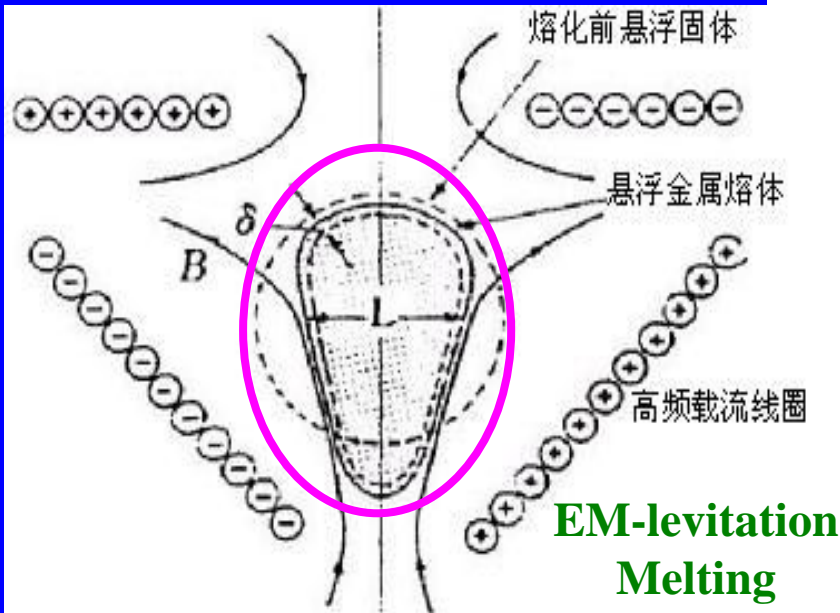
(in a continuously changing state)

## The Force Balance on a Free Surface of Alloy Melt:

表面张力 + 熔体的静、动压力 + 电磁力(Lorentz力) = 0



Two types of Cold Crucible Processes

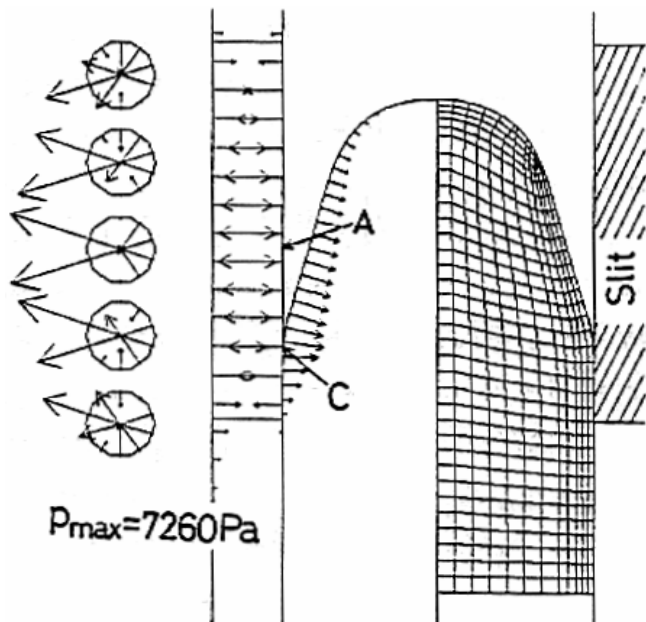


EM-levitation Melting

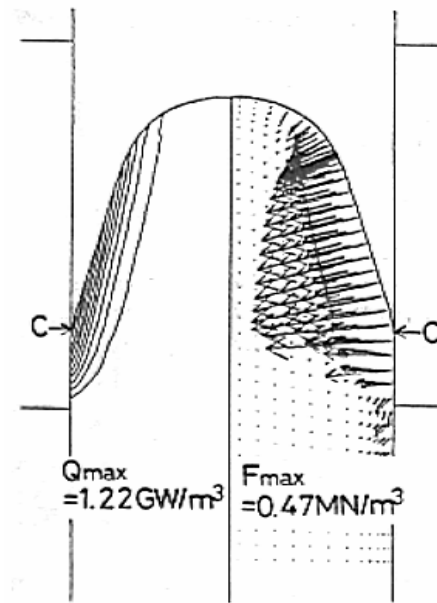
Fully numerical description for a 3-D free surface morphology of alloy melt in a dynamically changing state is a stiff task for the modelers in materials metallurgy and solidification fields.

“ **Liquid Metal Flow with Heat Transfer in a Cold Crucible  
Confined by a Free Surface and a Solidification Front**”,

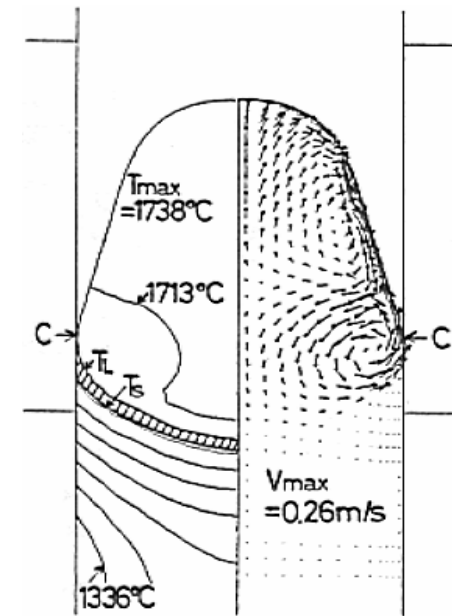
by T. TANAKA *et al*, *ISIJ International*, 1991, v.31, pp.1416-1423.



Configuration and mesh used  
for calculating the fields of  
velocity and temperature



Calculated contours  
of Joule heat and  
magnetic body force



Calculated fields of  
temperature and  
velocity in a Melt