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ANALYZING A MODULATED ELECTROMAGNETIC m=2 FORCING AND ITS CAPABILITY TO SYNCHRONIZE THE LARGE SCALE CIRCULATION IN A RAYLEIGH-BÉNARD CELL OF ASPECT RATIO Γ = 1

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Abstract: The synchronization of the helicity of an instability with azimuthal wavenumber m = 1 by a weak tidal m = 2 perturbation might play a key role in explaining the phase-stable Schwabe cycle of the solar dynamo [1]. In order to elucidate this type of interaction, we study a thermally driven Rayleigh-Bénard Convection (RBC) under a tide-like influence. We focus first on the generation of the m = 2 mode flow by an electromagnetic forcing, and second on its low-frequency modulation. In the last part, we present preliminary results on the interaction of this perturbation with the sloshing/torsional motion of the Large Scale Circulation (LSC) of an RBC flow. While the main focus of the paper is on the numerical side, some comparisons with experimental results are also made.

Key words: Magnetohydrodynamics, Rayleigh-Bénard convection, liquid metal flow, electromagnetic forcing, CFD

1. Introduction

In a series of recent papers [1-3], an attempt was made to explain the surprising phase stability of the solar Schwabe cycle in terms of the synchronization of the helicity of some instability with azimuthal wavenumber m = 1 by the m = 2 spring tides of the tidally dominant planets Venus, Earth and Jupiter. While, in the very solar context, the Tayler instability [4,5] and magneto-Rossby waves are the most promising representatives of the underlying m = 1 mode, the m = 1 Large Scale Circulation (LSC) of Rayleigh-Bénard Convection (RBC) [6] might be better suited for an experimental demonstration of this generic synchronization mechanism. The focus of this paper will be on the question of whether and how the helicity oscillation, which is connected with the typical sloshing and torsional motions of the LSC, can be synchronized by a tide-like perturbation.

2. Numerical model

In a corresponding experiment to be numerically simulated [7], we use a cylindrical container, completely filled with the low Prandtl number eutectic liquid metal GaInSn, with a diameter of D=0.18 m and a height of H=0.18 m. The LSC in this aspect ratio $\Gamma = \frac{D}{H} = 1$ consists of a single roll. To generate the electromagnetic forcing in the fluid, we use two coils with an inner height of 350 mm and an inner width of 100 mm of the MULTIMAG system [8].

Each one consists of 80 windings. They are situated on opposite sides of the container with an average distance from the x-z plane of 285 mm. At the bottom and top of the container, two thermally controlled copper plates (D = 220 mm, H = 25 mm) are installed to generate the RBC (Fig. 1). These plates have a significant effect on the magnetic field and the resulting force structure, as will be shown further below.
The flow was computed with the open source code library OpenFOAM 6 by solving (in DNS) the incompressible Navier-Stokes equation and the continuity equation:

$$\rho\frac{\partial \mathbf{u}}{\partial t}+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}=-\nabla p+\mu\nabla^2\mathbf{u}+M(t)\cdot \boldsymbol{F}_{EM}\,, \tag{1}$$

$$\nabla\cdot\mathbf{u}=0\,, \tag{2}$$

where $\boldsymbol{u}$ denotes the flow velocity, p the pressure, μ the dynamic viscosity and ρ the fluid density. The time-constant electromotive force (averaged over one period of the basic excitation, with f=25Hz in our case),

$$\boldsymbol{F}_{EM}=(\boldsymbol{j}\times\boldsymbol{B}), \tag{3}$$

which was pre-computed by Opera 1.7, is modulated by the factor $M(t)$. The generated velocities are typically in the range of some mm/s, which is, on the one hand, significantly smaller than the motion of the applied AC magnetic field with characteristic speed $2\cdot\pi\cdot f\cdot D = 2\cdot\pi\cdot 25\cdot 0.18$m/s = 28.3m/s. On the other hand, the low flow velocity leads also to a very small magnetic Reynolds number of the order of $10^{-3}...10^{-2}$. With these two facts taken together, in our specific case the driving force can be considered as basically flow-independent which justifies to decouple the electromagnetic calculations in Opera from the flow calculation in OpenFOAM.

As boundary condition of the flow field at the solid walls, the no-slip condition $\boldsymbol{u} = 0$ was implemented. To maintain a good proportion between calculation time and accuracy (Fig. 2) a mesh of 0.5×10^6 hexahedral cells, with contracted cells at the walls, was used.

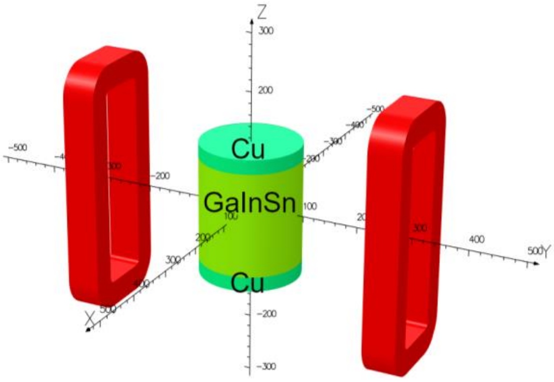

Fig. 1: Model setup for pre-computing the electromagnetic force density in Opera 1.7. The two coils (red) are used in our experiment [7].

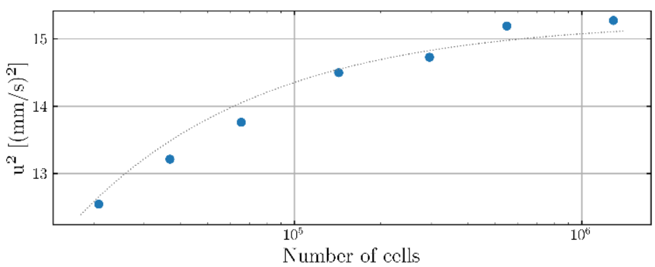

Fig. 2: Mesh convergence study (9.7 A and 25 Hz) using the time and space averaged value of u^2 for selected cell numbers. (blue) The gray dotted line represents $u^2=15.28(\mathrm{mm/s})^2\cdot(1-130\cdot N^{(-2/3)})$.

3. Results and Discussion

Fig. 3 illustrates the vector field of the electromagnetic body force for the two cases with and without copper plates at the top and the bottom of the container. The difference between the two cases is striking: while in both cases the vectors point essentially inward (in x-direction), the presence of copper plates homogenizes the body force in vertical direction. Without copper plates some part of the force field points from the top and bottom walls into the container. Accordingly, the presence of the copper plates fosters a tide-like m = 2 shaped force field.

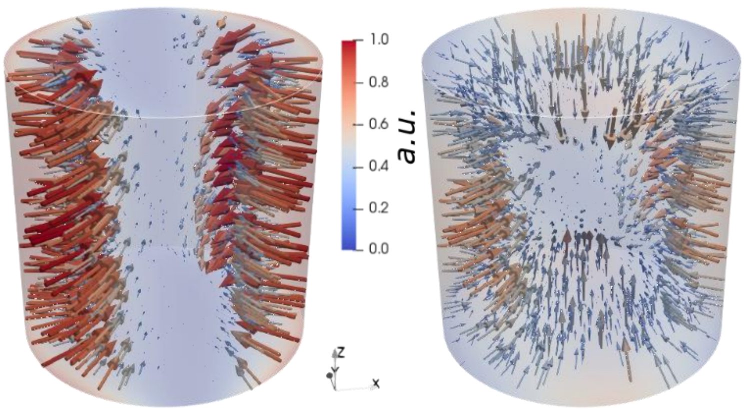

Fig. 3: Electromagnetic body force computed with (left) and without (right) 25 mm thick copper plates. The color code indicates the amplitude of the forec normalized to its maximum (with copper plates)

In a first step, the generic problem of flow generation in the liquid metal by a constant m = 2 electromagnetic forcing ($M(t) = 1$) was analyzed. To generate that flow in OpenFOAM, the body force $\boldsymbol{F}_{EM}$ was added as force density to the "PimpleFoam"-Solver. Depending on the strength of the applied body force to an $\boldsymbol{u} = 0$ m/s base state, a flow with a maximum velocity of a few mm/s could be created. For example, applying an AC current with an RMS value of 5.9 A and a frequency of 25 Hz to the coils creates a force density which can generate a flow up to a speed of 6 mm/s, while the combination of 9.7 A and 25 Hz provides flow velocities of up to 10 mm/s. After a certain time, the flow reaches a quasi-steady state, whose time averaged velocity field shows four quasi-two dimensional counter rotating vortices (Fig. 4).

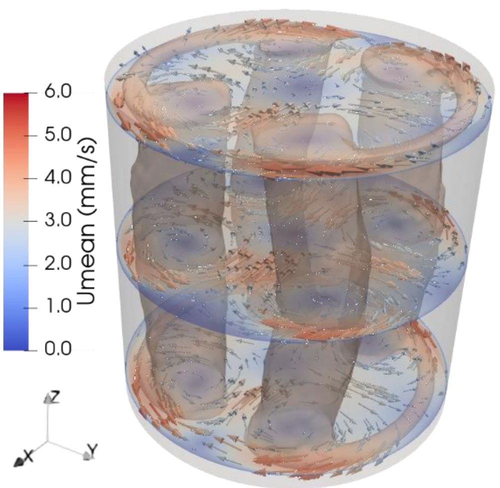

Fig. 4: Velocity field time averaged over 10^4 s, showing four quasi-two-dimensional vortices [7].

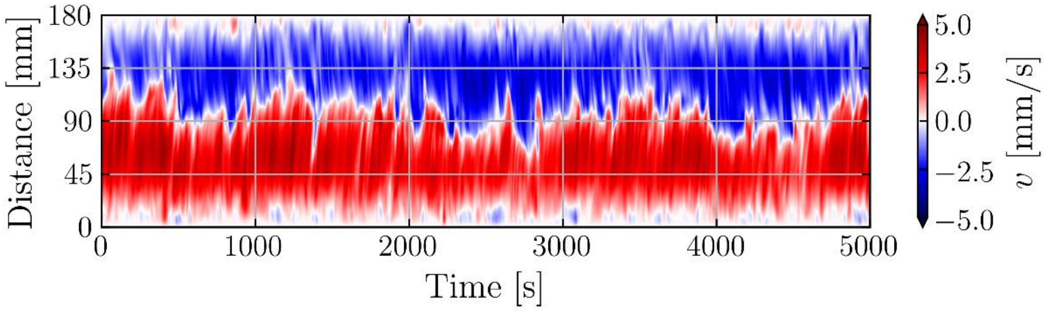

Fig. 5: Contour plot of the space-time distribution of the flow on the x-axis of the center plane for a coil-current with 5.9 A and 25 Hz.

The force field shown in the left plane of Fig. 3 produces a flow that mainly points radially inward along the x-axis forming two jets directed towards each other (Fig. 4). To close the circulation, two outward directed jets are formed on the y-axis. Due to the colliding jets in the center of the cylinder, strong fluctuations occur, and the instantaneous flow can significantly deviate from the averaged one. As an illustration, Fig. 5 shows the movement of the $\boldsymbol{u}$ = 0 m/s stagnation point along the x-axis around the center point. The distribution of this stagnation point shows a clear bi-modal behavior [7], mirroring the tendency to stay preferably on either side of the line rather than at the center. Yet, the radial velocity component v_r at the radius r = 0.5 R in the center-plane shows a clear dominance of the m = 2 mode, as inferred from its expansions into azimuthal wave numbers [7].

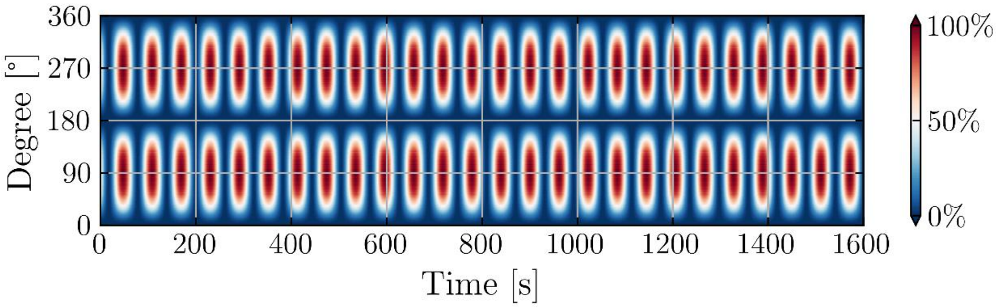

Fig. 6: Modulation of the body force over 360° of a circle with r = 0.9•R at the center plane

In a second step, the amplitude of body force $\boldsymbol{F_{EM}}$ was slowly modulated to emulate the tidal behaviour by a sinusoidal factor *M(t)*:

$$M(t) = sin^2(t \cdot f_{LSC} \cdot \pi) \quad (4)$$

Here, the dominant frequency f_{LSC} of the LSC in the RBC was specifically chosen to meet the resonance point, although a combination of electromagnetic forcing and RBC is not yet considered in this step (the dominant frequency f_{LSC} was just extracted from a “buoyantBoussinesqPimpleFoam” solved RBC for the temperature difference between top and bottom ΔT = 5 K, corresponding to Ra = 10^7).

The averaged flow velocity resulting from the modulated body force (with a coil-current of I = 5.9 A and f = 25 Hz) is about 6/7 of the unmodulated force field and exhibits also a slightly calmer behavior (Fig. 7), with fewer fluctuations. The mean flow velocities for modulated forcing and undisturbed RBC (with ΔT = 5 K) show a ratio of approximately 1/6 in the experiment and 1/7 in the simulation. These meets our demand for a weak m = 2 forcing, which does not overwhelm the basic RBC flow. A modal analysis in the azimuthal direction of the radial velocity component v_r at the specific radius r = 0.5 R in the center-plane shows a clearly dominant m = 2 mode, like in our first step. In a temporal Fourier analysis of the azimuthal m=2 mode a pronounced frequency, identical to the excitation frequency f_{LSC}, can be perceived. (Fig. 8).

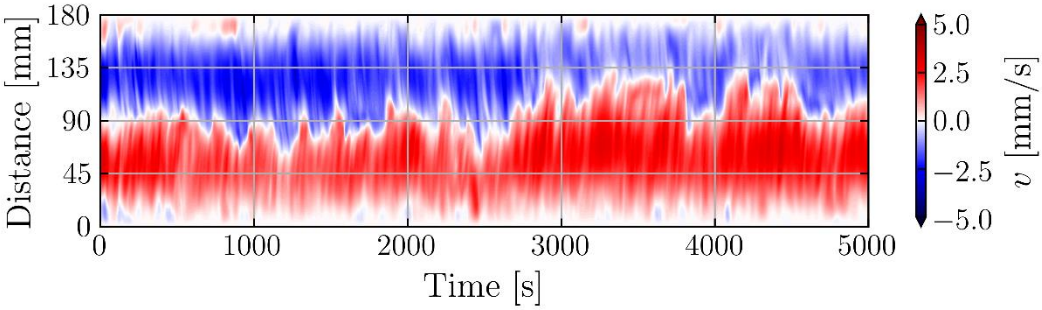

Fig. 7: Contour plot of the space-time distribution of the flow on the x-axis of the center plane for a time modulated body force (5.9 A and 25 Hz with modulation frequency of 0.0164 Hz)

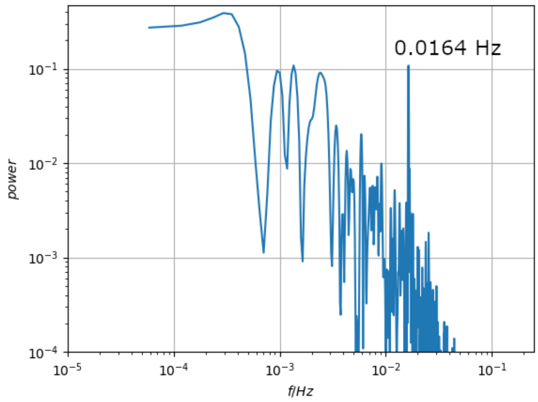

Fig. 8: m = 2 mode frequency spectrum of a flow generated by a modulated (f = 0.0164 Hz) body force

In a third step, some preliminary investigations on the combination of the modulated m = 2 tide-like body force with the m = 1 LSC of the RBC were carried out. The numerically determined oscillation frequency of the RBC at ΔT = 5 K (Ra = 10^7) is $f_{LSC} = 0.0168\,Hz$. This frequency, and its doubled value, are used as modulation frequencies of the term M(t) of the applied body force added to an established RBC. A first analysis of the LSC torsional angle in the temperature field shows that the application of the modulated body force regulates the oscillation for both considered modulation frequencies. (Fig. 9) A similar behavior was observed in the first experiments. In the long simulations (t_{SIM} = 7200 s) the dominant frequency

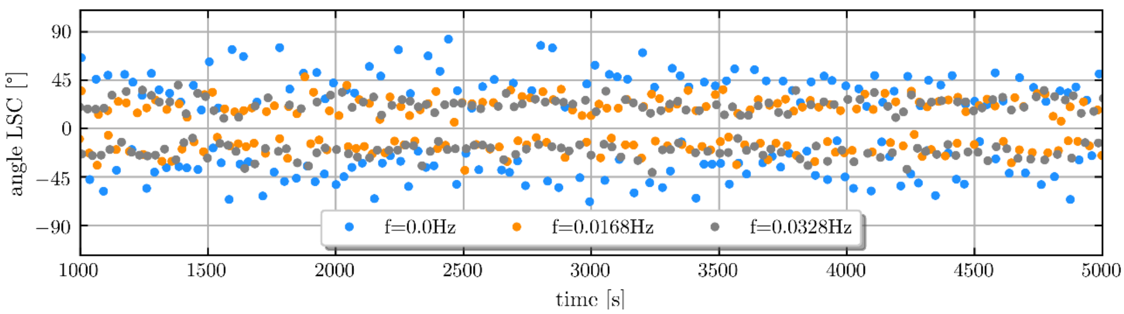

Fig. 9: Maximal and minimal torsion angles of the LSC in pure RBC and under the influence of modulated body forces.

f_{LSC} moves to slightly higher frequency of $f = 0.02 Hz$ for both modulation frequencies. Its main features are, however, comparable to those of the undisturbed RBC.

4. Conclusions

We have studied the generation of a tide-like m = 2 flow structure by means of an electromagnetic body force in a cylindrical volume filled with the liquid metal GaInSn. A low-frequency modulation of this force by a sinusoidal factor M(t) becomes clearly visible in the spectrum of the m = 2 mode. The resulting weak flow field is considered the right prerequisite for synchronizing the helicity of a (stronger) LSC of RBC, quite similarly as was shown previously for a m = 2 viscosity modulation in case of the Tayler instability [1,2].

Some first hints on the influence of the modulated m = 2 force onto the m = 1 LSC of a RBC were also obtained. This became particularly visible in a certain “regularization” of the torsional/sloshing motion. Yet, more numerical and experimental parameter studies will be needed to establish clearly the corresponding synchronization mechanism.

5. Acknowledgement

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