N-body simulation of the cosmic screening effect

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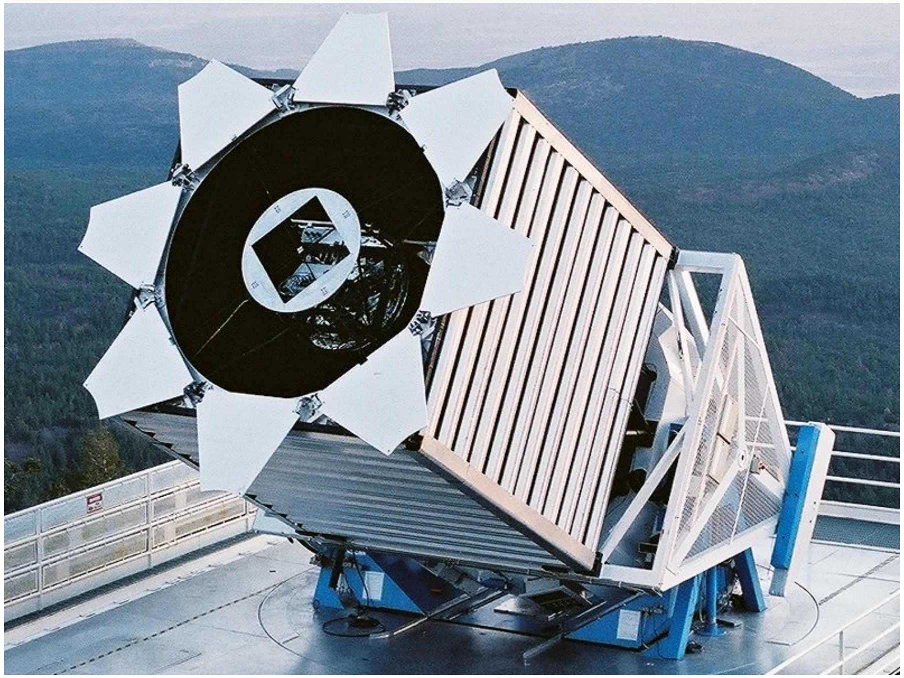
Due to modern telescopes, it was found that the Universe is filled with a cosmic web which is composed of interconnected filaments of galaxies separated by giant voids.

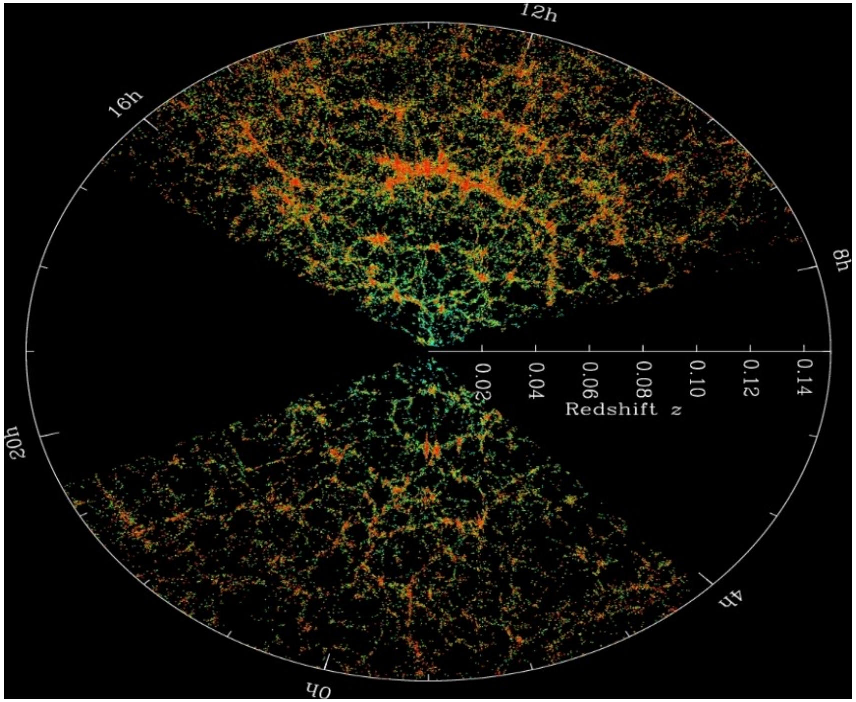
e.g.

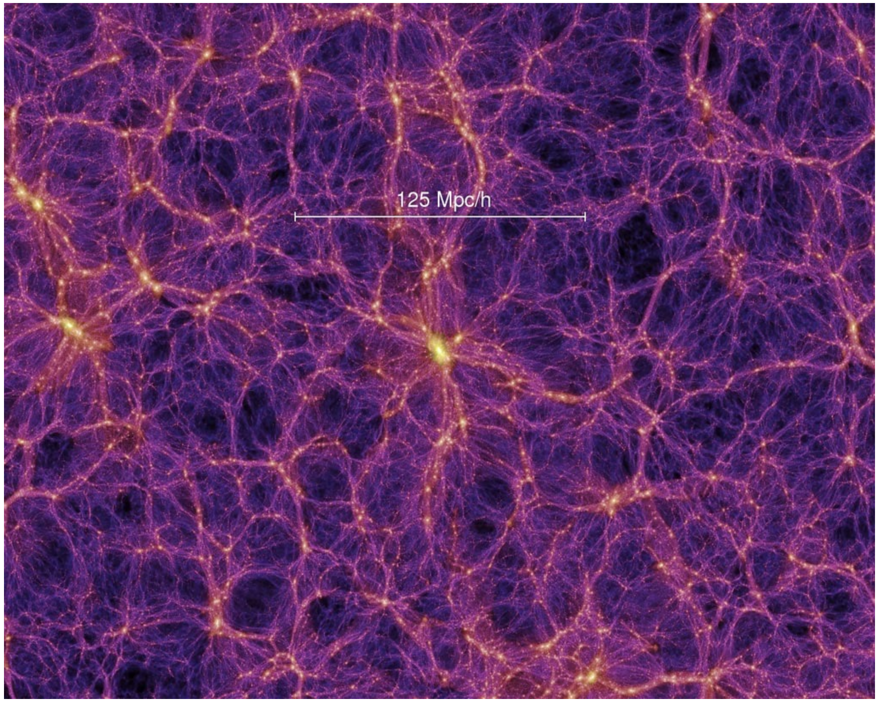
Sloan Digital Sky Survey (SDSS) 2.5-m wide-angle optical telescope at Apache Point Observatory in New Mexico, United States.



Sloan Digital Sky Survey:

N-body simulation:





The emergence of this large-scale structure (LSS) is one of the major challenges of modern cosmology.

Different approaches to studying LSS formation:

Analytical methods

Bardeen, Mukhanov, Rubakov&Gorbunov

works well in linear regime:

density contrast $|\delta\varepsilon / \varepsilon| < 1$

early stages of evolution of the Universe or large scales of the late Universe

Numerical simulation

works well in non-linear regime too:

density contrast may exceed unity

$$|\delta\varepsilon / \varepsilon| > 1$$

Gravity is the main force to form LSS!

Newtonian N-body simulations (e.g. GADGET-4)

Drawbacks:

- does not take relativistic effects (horizons, modification of gravitational interaction, …) that occur at large cosmological scales.
- not applicable for objects with relativistic peculiar velocities.
- problematic to apply the Newtonian approach to theories beyond the ΛCDM model.
- not appropriate for calculating the effect of backreaction of perturbations on the metric.

These drawbacks can be avoided in the framework of General Relativity.

⇓

Relativistic N-body cosmological simulation !

Two steps:

I. Derivation of the corresponding equations of motion.

Gravitational field is weak at all scales (with the exception of the vicinity of BH and NS) ⇒ **Theory of perturbations**

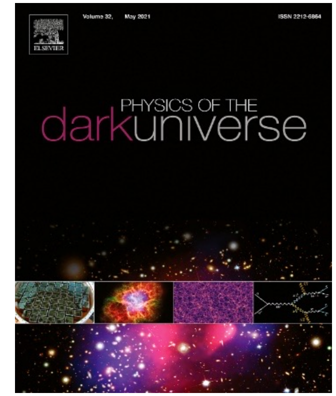
II. Creation of N-body cosmological simulation code.

Let us start from the first step!

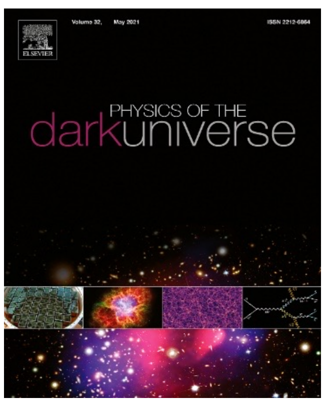
Cosmic screening approach

M. Eingorn, Astrophys. J. 825 (2016) 84

M. Eingorn, C. Kiefer, A. Zhuk, JCAP 09 (2016) 032



M. Eingorn, N.D. Guran, A. Zhuk, Phys. Dark Univ. 26 (2019) 100329



E. Canay, M. Eingorn, Phys. Dark Univ. 29 (2020) 100565

M. Eingorn, A.E. Yükselci, A. Zhuk, Phys. Lett. B 826 (2022) 136911

…

Cosmic screening approach

Theory of scalar (for simplicity!) perturbations

Perturbed FLRW metric:

Gravitational potential $\Phi << 1$

$$ds^2 = a^2\left[\left(1+2\Phi\right)d\eta^2 - \left(1-2\Psi\right)\delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

First order smallness perturbations
(for ideal perfect fluid $\Psi = \Phi$ **)**

Linearized Einstein eqs.:

$$(1) \quad \Delta\Phi - 3\tilde{\mathrm{H}}(\Phi' + \tilde{\mathrm{H}}\Phi) = \frac{1}{2}\kappa a^2 \delta T^0_{0(\mathrm{CDM})}$$

$$(2) \quad \frac{\partial}{\partial x^\beta}(\Phi' + \tilde{\mathrm{H}}\Phi) = \frac{1}{2}\kappa a^2 \delta T^0_{\beta(\mathrm{CDM})}$$

fluctuations of EMT of CDM

$$(3) \quad \Phi'' + 3\tilde{\mathrm{H}}\Phi' + \left(2\tilde{\mathrm{H}}' + \tilde{\mathrm{H}}^2\right)\Phi = 0$$

We consider CDM as a set of point-like inhomogeneities (e.g. galaxies, groups and clusters of galaxies)

Energy-momentum tensor (EMT) of inhomogeneities (e.g. Landau&Lifshitz):

$$T^{\mu\nu} = \sum_n \frac{m_n c^2}{(-g)^{1/2}[\eta]} \frac{dx_n^\mu}{d\eta} \frac{dx_n^\nu}{d\eta} \frac{1}{ds_n/d\eta} \delta(\mathbf{r} - \mathbf{r}_n)$$

$$\tilde{v}_n^\alpha \equiv \frac{dx_n^\alpha}{d\eta} = \frac{a}{c}\frac{dx_n^\alpha}{dt} = \frac{av_n^\alpha}{c} = \frac{v_{ph\,n}^\alpha}{c}, \quad \alpha = 1,2,3 -$$ **comoving peculiar velocity**

$$\rho = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n) \equiv \sum_n \rho_n -$$ **comoving mass density**

Weak field approximation: $\Phi << 1$. **However,** $\delta\rho / \bar{\rho}$ **can be** $\gg 1$

Due to explicit dependence T_0^0 on g_{ik}

Nonlinearity of GR!
Relativistic effect!

$$\delta T^0_{0(\mathrm{CDM})} \equiv \delta\varepsilon \approx \frac{\delta\rho c^2}{a^3} + \frac{3\bar{\rho}c^2}{a^3}\Phi$$

Comoving mass density fluctuations:

$$\delta\rho = \rho - \bar{\rho}$$

$$\rho = \sum_n \rho_n = \sum_n m_n \delta\left(\vec{r} - \vec{r}_n\right), \qquad \bar{\rho} = \mathrm{const}$$

Fluctuation of the mass density can be much bigger than its constant average value: $\delta\rho \gg \bar{\rho}$ **!**

⇓

Our approach works at all scales (from relatively small astrophysical scales to large cosmological ones)

Scalar perturbations in ΛCDM:

$$(1) \quad \Delta\Phi - 3\tilde{H}(\Phi' + \tilde{H}\Phi) = \frac{1}{2}\kappa a^2 \delta\varepsilon = \frac{1}{2}\kappa a^2 \left(\frac{\delta\rho c^2}{a^3} + \frac{3\bar{\rho} c^2}{a^3}\Phi \right)$$

$$(2) \quad \Phi' + \tilde{H}\Phi = -\frac{1}{2}\kappa a^2 \bar{\varepsilon} v = 0$$

If we neglect the peculiar velocities

Peculiar velocity potential

$$\Rightarrow \quad \boxed{\Delta\Phi - \frac{a^2}{\lambda^2}\Phi = \frac{\kappa c^2}{2a}\delta\rho}$$

Helmholtz Equation!

Screening length:

$$\lambda = \left[\frac{3\kappa}{2}\bar{\varepsilon}\right]^{-1/2} = \frac{1}{\sqrt{3(1+q)}}\frac{c}{H}$$

ΛCDM: $\kappa c^2\bar{\varepsilon} = 2(1+q)H^2$

$$= \sqrt{\frac{2a^3}{3\kappa\bar{\rho}c^2}} \sim a^{3/2}$$

$\bar{\rho} \to 0$ ∞

At the present time:

$$\lambda_0 \approx 3.7 \times 10^3 \mathrm{Mpc}$$

Solution:

$$\Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\vec{r}-\vec{r}_n|}\exp\left(-\frac{a}{\lambda}|\vec{r}-\vec{r}_n|\right)$$

Effect of the peculiar velocities

(*Canay, Eingorn*, Phys. Dark Univ. 29 (2020) 100565):

$$\Phi = \frac{1}{3}\left(\frac{\lambda_{\rm eff}}{\lambda}\right)^2 - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\vec{r}-\vec{r}_n|}\exp\left(-\frac{a}{\lambda_{\rm eff}}|\vec{r}-\vec{r}_n|\right)$$

$$\lambda \to \lambda_{\rm eff} = \sqrt{\frac{c^2 a^2 H}{3}\int_0^a \frac{da}{a^3 H^3}}, \qquad \frac{1}{3} \to \frac{1}{3}\left(\frac{\lambda_{\rm eff}}{\lambda}\right)^2$$

During the matter-dominated stage: $\lambda_{\rm eff} \propto a^{3/2}$

For the standard ΛCDM model at the present time:

$$\lambda_{\rm eff\,0} \approx 2.57\,\mathrm{Gpc} \approx 8.38\times 10^9\,\mathrm{ly} \approx 7.93\times 10^{27}\,\mathrm{cm}$$

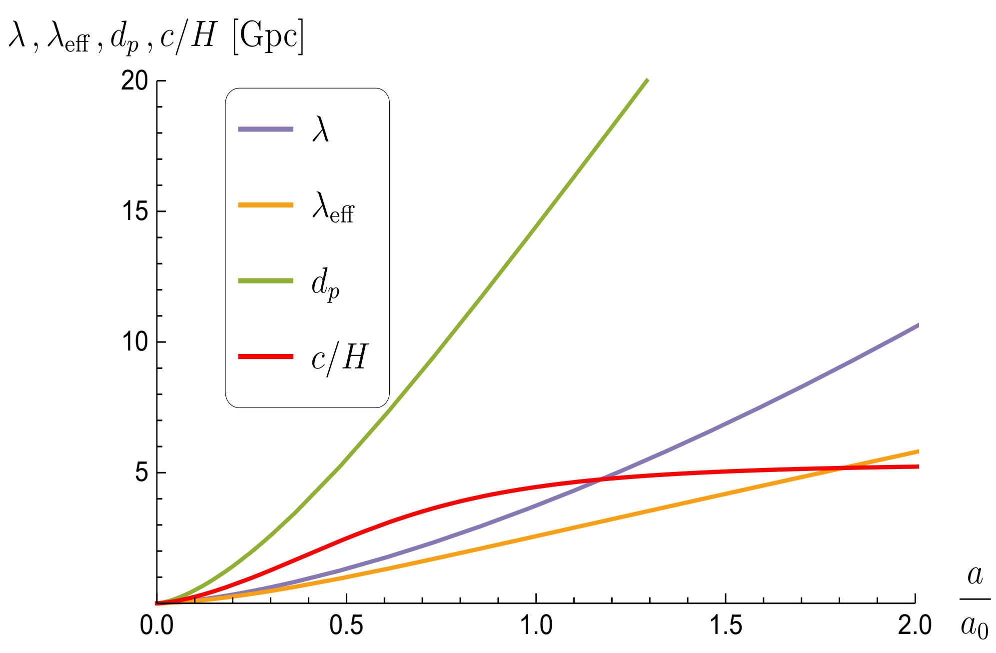
The Yukawa interaction ranges $\lambda, \lambda_{\rm eff}$ and the horizons:

Hubble horizon: $c/H_0 \approx 4.45\,\mathrm{Gpc} > \lambda_0, \lambda_{\mathrm{eff},0}$

Particle horizon: (This is the farthest distance that any photon can freely stream from the Big Bang – the size of the observable Universe)

$$d_p(t_0) = a(t_0)\int_0^{t_0}\frac{c\,d\tilde{t}}{a(\tilde{t})} \approx 14.42\,\mathrm{Gpc}$$ **-radius of the observable Universe**

Number of Yukawa regions: $d_p^3(t_0)/\lambda^3_{\mathrm{eff},0} \approx 177$



Intersection points in ΛCDM:

$$\lambda = c/H \text{ at } a \approx 1.17 a_0$$

$$\lambda_{\rm eff} = c/H \text{ at } a \approx 1.81 a_0$$

The gravitational interaction undergoes an exponential cut-off at distances

$$R \gtrsim \lambda_{\rm eff}$$

⇓

Matter overdensities stop growing on this cosmological scales.

⇓

Upper bound to the size of individual cosmic structures, like walls and filaments, in favour of the Cosmological Principle!

Cosmic screening provides a theoretical basis for the Cosmological Principle.

NOTE: The largest structure in the Universe is

Great GRB Wall (Hercules-Corona-Borealis Great Wall)

$$l \sim 2 - 3\,\text{Gpc}$$

A region of the sky seen in the data set mapping of gamma-ray bursts (GRBs) that has been found to have an unusually higher concentration of similarly distanced GRBs than the expected average distribution

supernova explosion followed by black hole formation

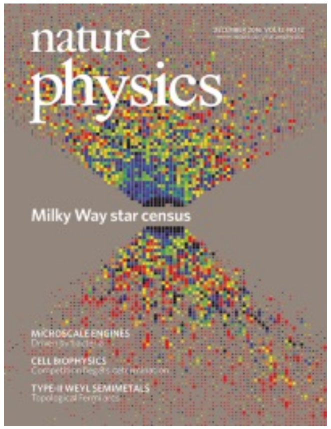
Effect of cosmic screening:

At the present time, the largest structures should be less than

$$\lambda_{\text{eff}0} \approx 2.6\ \text{Gpc}$$

Numerical confirmation of the cosmic screening effect

N-body simulation code:
modified relativistic code gevolution



Nature Phys. 12 (2016) 346;

J. Adamek, D. Daverio, R. Durrer and M. Kunz

The equations in this code include not only linear terms, but also those which are quadratic in scalar perturbations. As a result, metric corrections represent <u>mixtures of the first- and second-order quantities.</u>

⇓

Mixing of orders of smallness leads to a rather complicated form of equations.

Cosmic screening approach

- **Orders of smallness are not mixed.**
- **The first order quantities are sources for the second order ones.**
- **All equations are linear** ⟹ **analytic solution in the case** ΛCDM
- **The cosmic screening effect is clearly manifested.**

The only limitations: $\Psi << 1, \quad v/c << 1$ ← not a mandatory condition

M.Brilenkov, E.Canay, M.Eingorn
European Phys. Journ. C (2023)

Simulations

M.Eingorn, E. Yukselci, A.Zhuk, Phys. Lett. B 826 (2022) 136911.

With the help of the corresponding alternative computer codes, we calculate the power spectra of $\Phi,\ \Phi-\Psi$ and $\vec{B}$ in gevolution and screening approaches and compare the results.

We have conducted a series of cosmological N-body simulations in boxes of sizes 280, 336, 560, 980, 1680 Mpc/h with 1 Mpc/h resolution as well as an additional series in boxes of sizes 280, 560, 1120, 2016, 2800 Mpc/h with 2 Mpc/h resolution

$$N = 1680^3 = 4\,741\,632\,000$$ **particles**

⇓

Supercomputer: National Center for High Performance Computing of Turkey (ITU, Istanbul)

Power spectra

Two-point correlation function:

$$\xi\left(\vec{r} \equiv \vec{x} - \vec{x}'\right) \equiv \left\langle \delta\left(\vec{x}\right)\delta\left(\vec{x}'\right)\right\rangle = \frac{1}{V}\int d^3x\,\delta\left(\vec{x}\right)\delta\left(\vec{x}-\vec{r}\right)$$

Fourier transform:

$$\xi\left(r\right) = \int \frac{d^3k}{\left(2\pi\right)^3}\tilde{P}\left(k\right)e^{i\vec{k}\left(\vec{x}-\vec{x}'\right)}$$

In the Fourier space:

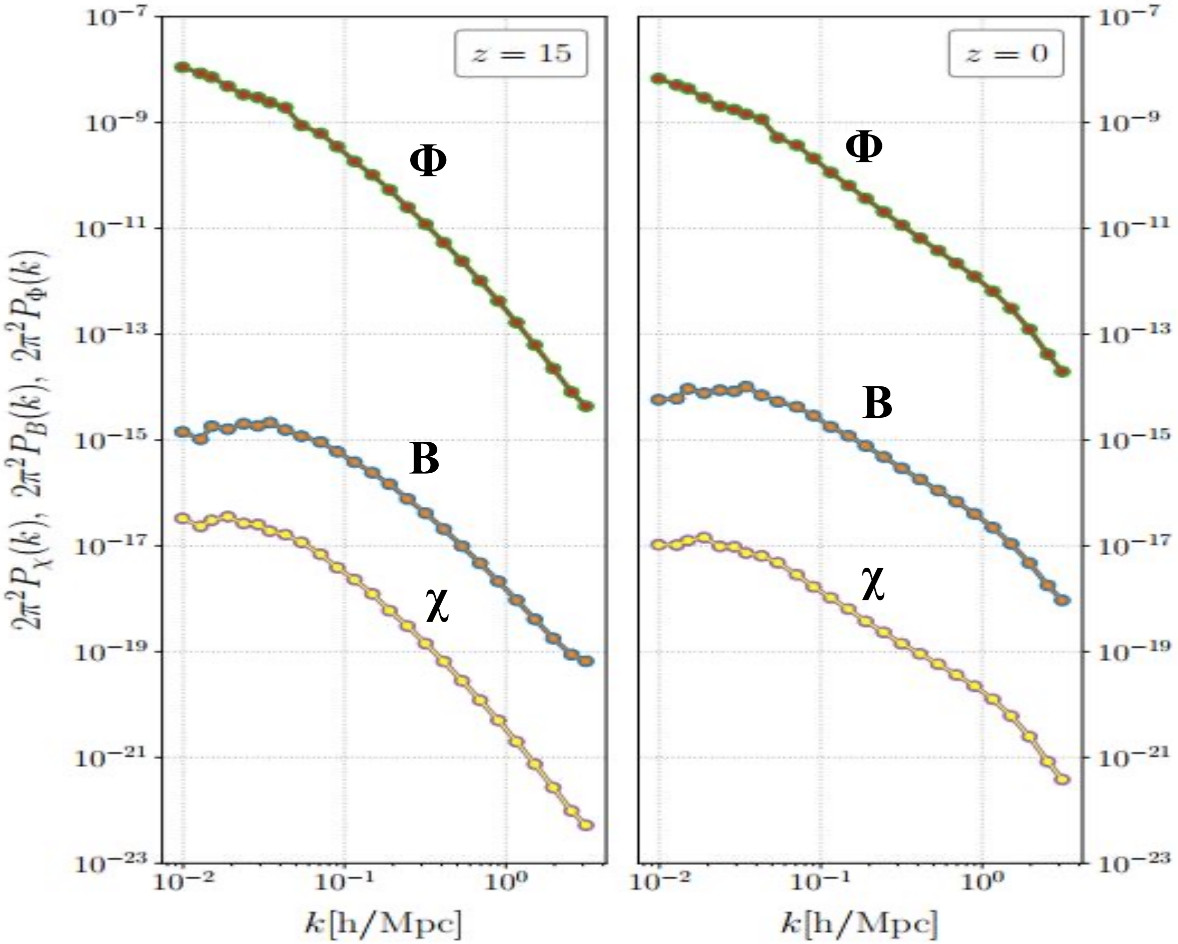
$$\left\langle \hat{\delta}\left(\vec{k}\right)\hat{\delta}\left(\vec{k}'\right)\right\rangle \equiv \left(2\pi\right)^3\tilde{P}\left(k\right)\delta_{\mathrm{D}}\left(\vec{k}-\vec{k}'\right) \equiv \frac{2\pi^2}{k^3}P\left(k\right)\delta_{\mathrm{D}}\left(\vec{k}-\vec{k}'\right)$$

Commoving momentum and commoving distance:

$$l = 2\pi / k, \qquad l_{\mathrm{ph}} = a\,l$$

Power spectrum shows the distribution of physical quantities at different scales.

Simulation box of commoving size 980 Mpc/h



Remarkable coincidence

!

$6.4\times10^{-3}\,\mathrm{h/Mpc} \le k \le \pi\,\mathrm{h/Mpc}$

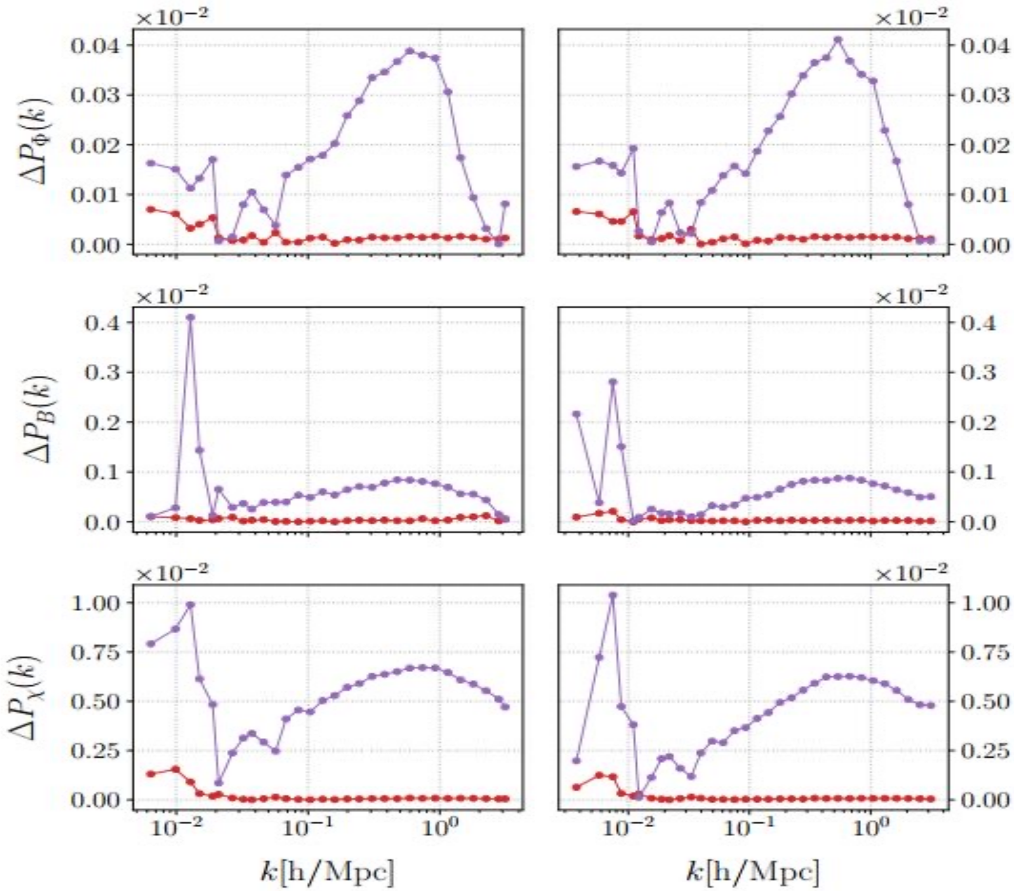
Power spectra of Φ (top curves), B (middle curves) and χ (bottom curves) from the "gevolution" code (green, blue, purple curves in the background) and from the "screening" code (red, orange, yellow curves in the foreground at redshifts z = 15 (left graph) and z = 0 (right graph).

Relative deviations

$$\Delta P \equiv \left| \frac{P_{\text{screening}} - P_{\text{gevolution}}}{P_{\text{gevolution}}} \right|$$

980Mpc/h 1680Mpc/h

$z = 0$



Φ

B

χ

$\Delta P_{\Phi} \leq 0.04\%$

$\Delta P_{B} \leq 0.4\%$

$\Delta P_{\chi} \leq 1\%$

Relative deviations of the power spectra of Φ, B and χ predicted by the "screening" code from the "gevolution" code counterparts at redshifts z = 15 (red) and z = 0 (purple).

- Despite the fact that the "**gevolution**" quantities **Φ** and **B** have the second-order admixtures, we have demonstrated that the power spectra are in very good agreement between the compared schemes. For example, the relative difference of the power spectra for **Φ** is 0.04% maximum. Hence, the effect of the second-order admixtures is small, as it should be.

- We have shown that the simpler "**screening**" code saves almost 40% of CPU hours.

cheaper the project cost

larger box for the fixed allotted time

Power spectrum of the mass density contrast

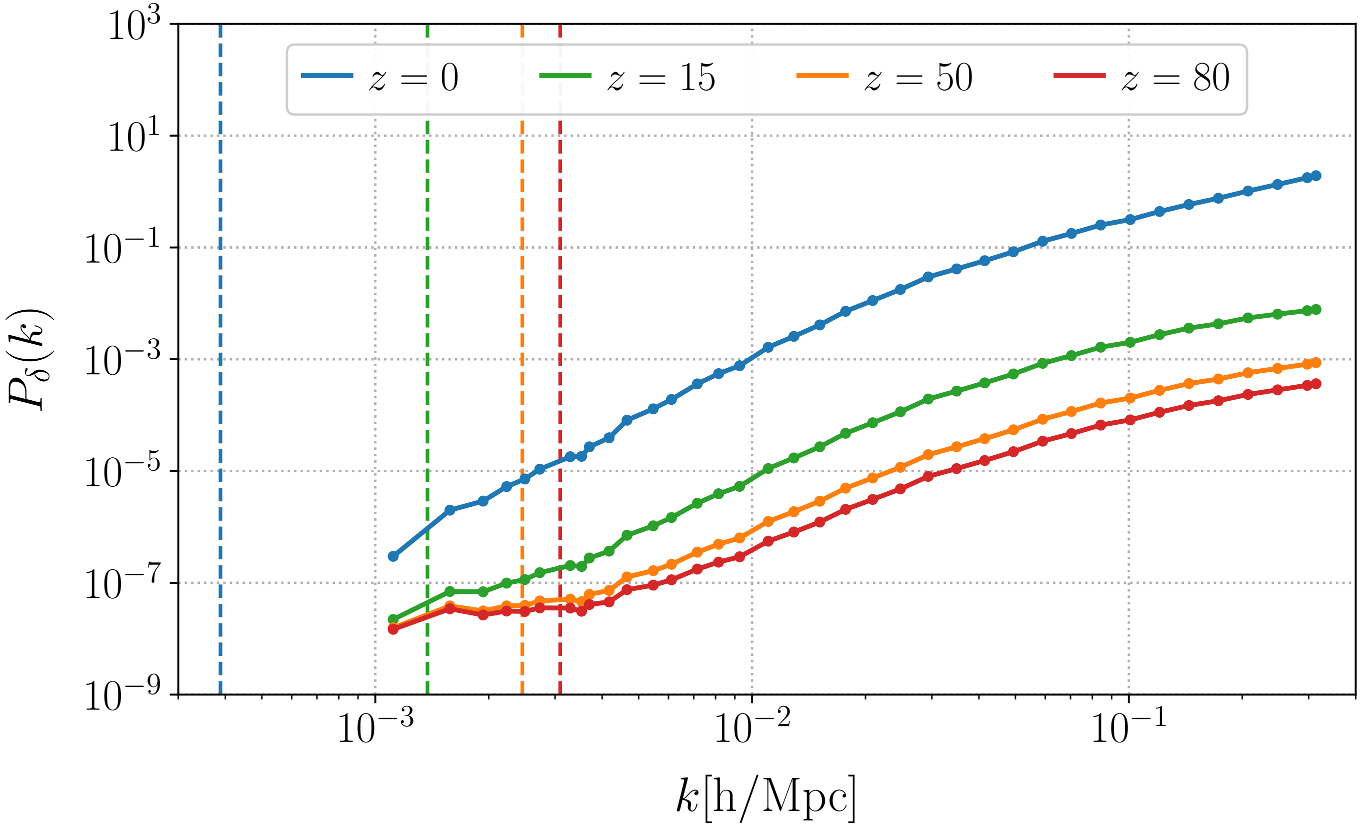
$$\delta \equiv \delta\rho/\bar{\rho} = (\rho - \bar{\rho})/\bar{\rho}$$

$$4\pi k^3 \langle \widehat{\delta}(\mathbf{k}, z)\, \widehat{\delta}(\mathbf{k}', z) \rangle = (2\pi)^3 \delta_{\rm D}(\mathbf{k} - \mathbf{k}') P_\delta(k, z)$$

Alternative definition:

$$\tilde{P}_\delta(k, z) = (2\pi^2/k^3) P_\delta(k, z)$$

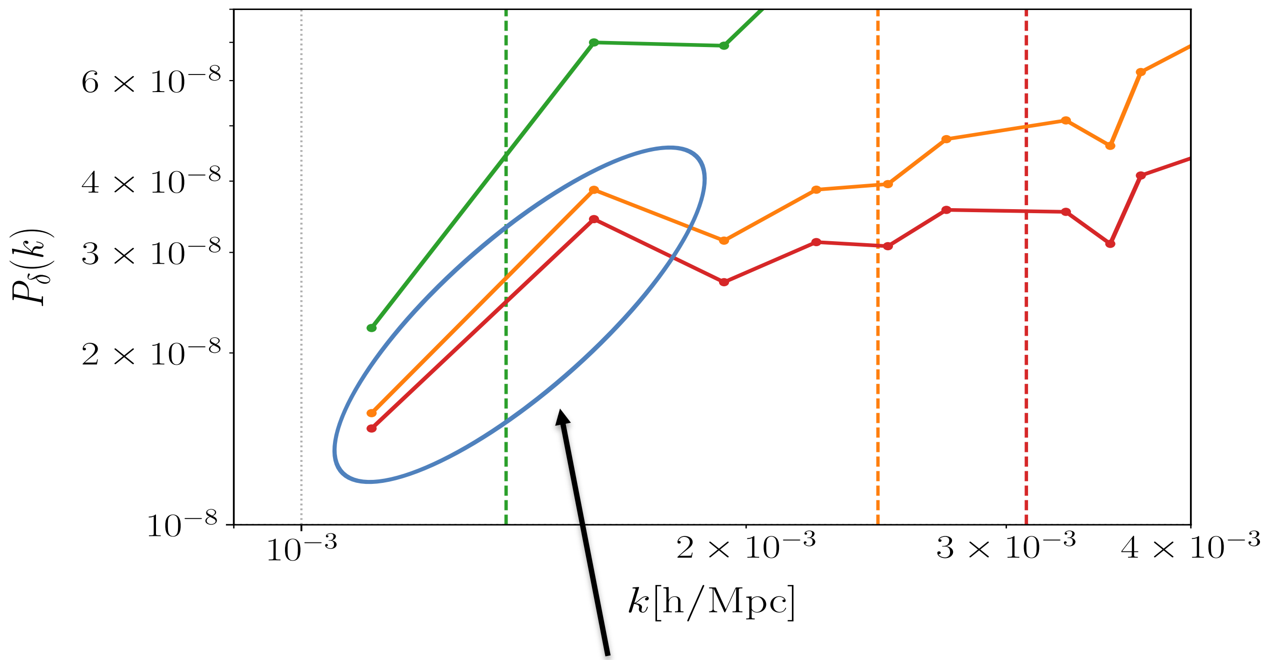
N-body simulation in the box with comoving size **5.632** Gpc/h in the cosmic screening approach. The resolution is **1** particle per **2** Mpc/h.



Vertical lines show the comoving momenta corresponding to the screening length.

$$k_{\mathrm{scr}}(z) = a/\lambda_{\mathrm{eff}}(z)$$
$$= 1/[(1+z)\lambda_{\mathrm{eff}}(z)]$$

Manifistation of the screening effect:



Orange and red curves converge towards each other.

⇩

Suppression of the growth of the density contrast with time on scales byond the screening length.

Correspondence with hydrodynamic approximation

large cosmological scales ⟺ **small values of *k***

Canay, Eingorn, Phys. Dark Univ. 29 (2020) 100565

On the MD stage:

$$\widehat{\delta\rho} \propto k^2\widehat{\phi}\left(a + \frac{5\kappa\bar{\rho}c^2}{2k^2}\right) = k^2\widehat{\phi}a_0\left(\frac{1}{1+z} + \frac{5}{3}\frac{k_0^2}{k^2}\right)$$

1. $k << \sqrt{5(1+z)/3}\,k_0 \equiv K_0(z) \approx \frac{a}{\lambda_{\mathrm{eff}}}$ ⟹ $\widehat{\delta\rho} \neq f(z)$

⇓

cosmic screening!

2. $k >> K_0$ ⟹ $\widehat{\delta\rho}(z_1)/\widehat{\delta\rho}(z_2) \approx (1+z_2)/(1+z_1) \neq f(k)$

⇓

Power spectra are parallel to each other

CONCLUSION

N-body simulation of the power spectra of the mass density contarst demonstrates the suppression of the growth of the density contrast with time on scales beyond the screening length.

This is a clear manifistation of the cosmic screening effect!

THANK YOU!