Backreaction of cosmological perturbations

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Cosmological Principle:

A sufficiently large volume, chosen arbitrarily in the Universe, should contain approximately the same amount of matter.

scale of homogeneity: 70 – 370 Mpc ???

Hercules-Corona Borealis Great Wall ~ 2-3 Gpc !

At scales larger than the scale of homogeneity, the averaged evolution of the Universe is governed by the FLRW metric

$$ds^2 = a^2(\eta)\left[d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta\right],$$

with the Friedmann equations (ΛCDM model)

$$\frac{3\mathcal{H}^2}{a^2} = \kappa\bar{\varepsilon} + \Lambda \qquad \textbf{and} \qquad \frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} = \Lambda$$

Small-scale inhomogeneities, e.g. galaxies and groups of galaxies, perturb the averaged metric:

$$ds^2 = a^2(\eta)\left[\left(1 + 2\Phi + 2\Phi^{(2)}\right) d\eta^2 - \left(1 - 2\Psi - 2\Psi^{(2)}\right)\ \delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

First-order perturbations

Second-order perturbations

$$\Phi = \Psi$$

Average values:

$$\overline{\Phi} = 0.$$

$\overline{\Phi^{(2)}}, \overline{\Psi^{(2)}}, \overline{\Phi^2}, \overline{\rho\Phi},$ ≠ 0 !!! **affect the averaged behavior of the Universe**

How strong?

In the case of **strong influece**:

1. The expansion of perturbations in terms of the degree of smallness (see above) is not correct.

2. The Friedmann equations must be modified.

Backreaction problem!

I. Backreaction in Friedmann equations

Within the cosmic screening approach, the perturbed Friedmann eqs (ApJ 845 (2017) 153):

$$\frac{3\mathcal{H}^2}{a^2} - \frac{6\mathcal{H}}{a^2}\left[\mathcal{H}\overline{\Phi^{(2)}} + \overline{(\Psi^{(2)})'}\,\right] - 3\kappa\overline{\varepsilon}\,\overline{\Psi^{(2)}} = \kappa\overline{\varepsilon} + \Lambda + \underline{\kappa\overline{\varepsilon}^{(\mathrm{II})}}\,,$$

$$\frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} - \frac{2}{a^2}\left[\,\overline{(\Psi^{(2)})''} + 2\mathcal{H}\overline{(\Psi^{(2)})'} + \mathcal{H}\overline{(\Phi^{(2)})'} + (2\mathcal{H}' + \mathcal{H}^2)\overline{\Phi^{(2)}}\,\right] = \Lambda - \underline{\kappa\overline{p}^{(\mathrm{II})}}\,,$$

effective average energy density and pressure:

$$\kappa\overline{\varepsilon}^{(\mathrm{II})} = \frac{\kappa}{2}\overline{\varepsilon}\frac{\overline{\rho\Phi_0}}{\overline{\rho}} - 5\left(\kappa\overline{\varepsilon} + \Lambda\right)\overline{\Phi_0^2}$$

$$\kappa\overline{p}^{(\mathrm{II})} = \frac{\kappa}{6}\overline{\varepsilon}\frac{\overline{\rho\Phi_0}}{\overline{\rho}} - \left(\frac{11}{6}\kappa\overline{\varepsilon} - \frac{5}{3}\Lambda\right)\overline{\Phi_0^2}\,.$$

$$\frac{\overline{\rho\Phi_0}}{\overline{\rho}},\quad \overline{\Phi_0^2}\quad \text{- ?}$$

The first-order velocity-independent potential **(ApJ 825 (2016) 84) :**

$$\Phi_0 = \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\mathbf{r}-\mathbf{r}_n|}\exp\left(-\frac{a|\mathbf{r}-\mathbf{r}_n|}{\lambda}\right)$$

The screening lengt:

$$\lambda = \sqrt{\frac{2a^3}{3\kappa\overline{\rho}c^2}}\,.$$

Inhomogeneities are considered in the form of a system of separate nonrelativistic point-like particles with masses m_n and comoving radius-vectors r_n.

$$\overline{\rho\Phi_0} = \frac{1}{3}\overline{\rho} - \frac{\kappa c^2}{8\pi a}\frac{1}{\mathcal{V}}\sum_n\sum_{k\neq n}\frac{m_n m_k}{|\mathbf{r}_k-\mathbf{r}_n|}\exp\left(-\frac{a|\mathbf{r}_k-\mathbf{r}_n|}{\lambda}\right)$$

A toy model: all particles (galaxies) have the same masses and are located at the same distance l from each other.

$$\overline{\rho\Phi_0} = \frac{1}{3}\overline{\rho}\left[1 - \frac{1}{4\pi\tilde{\lambda}^2}\sum_{k_1=-\infty}^{+\infty}\sum_{k_2=-\infty}^{+\infty}\sum_{k_3=-\infty}^{+\infty}\frac{1}{\sqrt{k_1^2+k_2^2+k_3^2}} \times \exp\left(-\frac{\sqrt{k_1^2+k_2^2+k_3^2}}{\tilde{\lambda}}\right)\right],$$

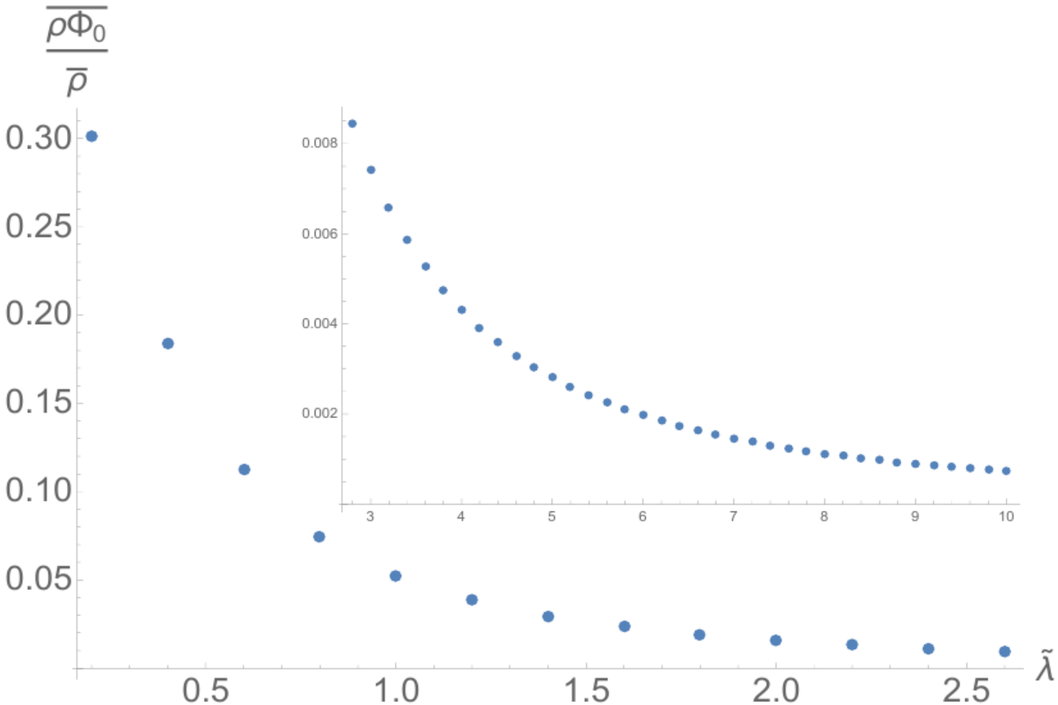

Figure 1: Behavior of $\overline{\rho\Phi_0}/\overline{\rho}$ as a function of the renormalized screening length $\tilde{\lambda}$.

Renormalized screening length:

$$\tilde{\lambda} = \frac{\lambda}{al} = \sqrt{\frac{2c^2}{9H_0^2\Omega_{\rm M}}}\,\frac{1}{a_0 l}\,\frac{1}{\sqrt{z+1}} \approx \frac{3740\,{\rm Mpc}}{a_0 l}\,\frac{1}{\sqrt{z+1}}\,,$$

$$a_0 l = 20\ {\rm Mpc}$$

$$z = 100\,,\quad \tilde{\lambda} \approx 18.6$$

$$z = 0\,,\quad \tilde{\lambda} \approx 187$$

$$z < 100,\quad \frac{\overline{\rho\Phi_0}}{\overline{\rho}} \ll 10^{-3}$$

Evaluation of $\overline{\Phi_0^2}$:

$$\overline{\Phi_0^2} = -\frac{1}{9} + \frac{\kappa c^2}{48\pi\overline{\rho}\lambda}\frac{1}{\mathcal{V}}\sum_n\sum_k m_n m_k \exp\left(-\frac{a|\mathbf{r}_k - \mathbf{r}_n|}{\lambda}\right)$$

$$\Rightarrow \quad \overline{\Phi_0^2} = -\frac{1}{9}\left[1 - \frac{1}{8\pi\tilde{\lambda}^3}\sum_{k_1=-\infty}^{+\infty}\sum_{k_2=-\infty}^{+\infty}\sum_{k_3=-\infty}^{+\infty}\exp\left(-\frac{\sqrt{k_1^2+k_2^2+k_3^2}}{\tilde{\lambda}}\right)\right]$$

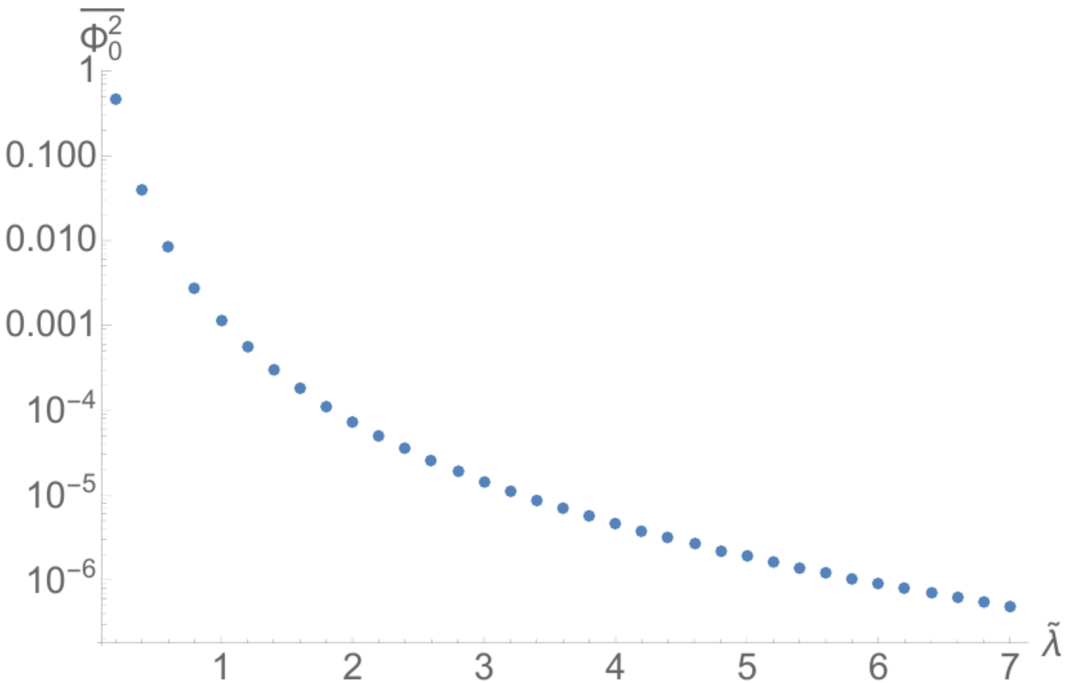

Figure 2: Behavior of $\overline{\Phi_0^2}$ as a function of the renormalized screening length $\tilde{\lambda}$.

$$z < 100 \Rightarrow \tilde{\lambda} > 18.6$$

$$\Downarrow$$

$$\Rightarrow \quad \overline{\Phi_0^2} << 10^{-6}$$

Conclusion 1:

The effective average energy density $\overline{\varepsilon}^{(\mathrm{II})}(\eta)$ and pressure $\overline{p}^{(\mathrm{II})}(\eta)$ have a negligible backreaction effect on the Friedmann equations.

II. Evaluation of the second-order term $\Psi_0^{(2)}$.

Phys.Dark Univ. 26 (2019) 100329:

$$\Psi_0^{(2)} = -\frac{3}{4}\Phi_0^2 + \frac{\Phi_0}{6} - \frac{\pi\overline{\rho}\lambda}{a}\left(\frac{\kappa c^2}{8\pi a}\right)^2 \sum_k m_k \; e^{-a|\mathbf{r}-\mathbf{r}_k|/\lambda}$$
$$+ \frac{1}{2}\left(\frac{\kappa c^2}{8\pi a}\right)^2 {\sum_{k,k'}}' m_k m_{k'} \frac{e^{-a|\mathbf{r}-\mathbf{r}_k|/\lambda}}{|\mathbf{r}-\mathbf{r}_k|} \frac{e^{-a|\mathbf{r}_{k'}-\mathbf{r}_k|/\lambda}}{|\mathbf{r}_{k'}-\mathbf{r}_k|},$$

$$\Longrightarrow \quad \Psi_0^{(2)} = -\frac{3}{4}\Phi_0^2 + \frac{\Phi_0}{6} - \pi\tilde{\lambda}\left(\frac{1}{12\pi\tilde{\lambda}^2}\right)^2 \sum_k e^{-|\tilde{\mathbf{r}}-\tilde{\mathbf{r}}_k|/\tilde{\lambda}}$$
$$+ \frac{1}{2}\left(\frac{1}{12\pi\tilde{\lambda}^2}\right)\left(\frac{1}{3} - \Phi_0\right) {\sum_q}' \frac{e^{-\tilde{r}_q/\tilde{\lambda}}}{\tilde{r}_q} \quad \Longrightarrow \quad -\frac{3}{4}\Phi_0^2, \quad \tilde{\lambda} >> 1$$

$$|\Phi_0| \ll 1 \quad \Longrightarrow \quad |\Psi_0^{(2)}| \ll |\Phi_0|.$$

Conclusion 2:

Therefore, the second-order correction $\Psi_0^{(2)}$ is much less than the first-order quantity Φ_0 as it should be!

Final conclusions:

1. The numerical evaluation shows that considered nonlinear perturbations have a negligible backreaction effect on the Friedmann equations.

2. The second-order correction to the gravitational potential is much less than the corresponding first-order quantity. Consequently, the expansion of perturbations into orders of smallness in the cosmic screening approach is correct.

THANK YOU!