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# The tensor analyzing power $T_{20}$ in deuteron break-up reactions within the Bethe-Salpeter formalism

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## Abstract

The tensor analyzing power  $T_{20}$  and the polarization transfer  $\kappa$  in the deuteron break-up reaction  $Dp \rightarrow pX$  are calculated within a relativistic approach based on the Bethe-Salpeter equation with a realistic meson-exchange potential. Our results on  $T_{20}$ ,  $\kappa$  and the cross section are compared with experimental data and non-relativistic calculations and with the outcome of a relativization procedure of the deuteron wave function.

## 1. Introduction

Deuteron break-up reactions  $A(D, p)X$  at medium and high energies are thought to serve as a source of information on the high momentum components of the deuteron wave function. In the impulse approximation (IA) the differential cross section is proportional to the deuteron single-nucleon momentum density. This gives hope for the possibility of a direct extraction of the deuteron wave function from the experiment. Numerous experimental data from Dubna [1] and Saclay [2] have confirmed the IA picture for small internal momenta of nucleons in the deuteron, while a reasonable description of the data at higher momenta requires the incorporation of relativistic effects. It has been shown that even a minimal relativization procedure [3,4] of the non-relativistic wave function is sufficient to describe fairly well the data in the kin-

ematical region measured so far, except for a relatively broad shoulder around  $p' \approx 0.3 \text{ GeV}/c$  ( $p'$  is the momentum of the final proton in the deuteron rest frame, see below). Some unconventional processes have been considered as small corrections to IA in attempts to explain this shoulder in the cross section [5]. The success of such a simple but descriptive interpretation of the data has allowed one to introduce the notion of an experimental deuteron wave function, often referred to the Moscow wave function [1,2], and to discuss different aspects of the deuteron structure in these experiments.

In the same experiments one measures the polarization characteristics of the deuteron, such as the tensor analyzing power  $T_{20}$  and the polarization transfer  $\kappa$ . These quantities are more sensitive to the reaction mechanism and to the internal structure of the colliding particles and give more detailed information. The

first data on polarization phenomena has shown a discrepancy with theoretical predictions based on IA with Bonn or Paris light cone wave functions [4]. The main disappointing moment here is that, while the theoretical calculation predicts a change of the sign of  $T_{20}$ , the experimentally deduced values seem to remain negative in the whole interval of the measured momenta. Very recent data [6] has confirmed that  $T_{20}$  is still negative up to  $p' \sim 0.5$  GeV/c. In this case an interpretation of the data in terms of a direct extraction of the deuteron wave function becomes doubtful. This challenge stimulated investigations of other than IA mechanisms of interaction (cf. [5] and references therein) and possible manifestations of non-nucleonic degrees of freedom in the deuteron. All these additional contributions have been investigated as corrections to minimal relativization, and conclusions about the role of other mechanisms have been drawn relative to these results. Apart from the interest in other mechanisms, the investigation of relativistic effects in different approaches is still of a great importance. We would like to emphasize that the contributions of other (unconventional) mechanisms may be discussed only after a consistent relativistic calculation of the main process. Nowadays the most complete relativistic investigation of one-nucleon exchange diagrams has been performed in Ref. [7] for backward elastic  $pD$  scattering by solving numerically the Bethe-Salpeter (BS) equation with a realistic interaction. Several authors [8] studied the relativistic effects in the deuteron by considering the  $D \rightarrow NN$  vertex within different approximations of the exact BS equation and applying it to electromagnetic and hadron elastic scattering of the deuteron. Up to now a consistent relativistic calculation for the deuteron break-up reactions and polarization phenomena in these processes is still lacking.

In this paper we present a relativistic analysis of the deuteron tensor analyzing power  $T_{20}$ , polarization transfer  $\kappa$  and cross section in forward break-up reactions. A fully covariant expression for these quantities is obtained within the BS formalism. Results of numerical calculations, utilizing the recently obtained numerical solution of the BS equation with a realistic interaction, are compared with the available experimental data and with several theoretical approaches, such as non-relativistic and light cone calculations.

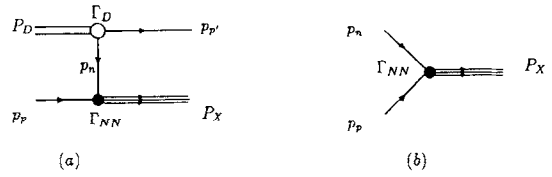


Fig. 1. The Feynman graphs for the  $Dp \rightarrow pX$ -reaction (a) and for the elementary reaction  $NN \rightarrow X$  (b).

## 2. The relativistic impulse approximation

We consider the inclusive break-up reaction of the type  $D + p \rightarrow p'(0^0) + X$ , where the typical initial energy of the polarized deuteron is in the order of a few GeV, and one final proton is detected in the forward direction. In a first approximation this reaction may be regarded as a process where one of the nucleons is removed from the deuteron by a small transferred momentum, while the other one continues to move nearly with the velocity it had before the collision. In this situation the cross section  $D + A \rightarrow p'(0^0) + X$  is even expected to be independent of the target  $A$ , and only the deuteron structure is relevant to describe the process. Such a simplified picture has been experimentally confirmed by detecting an almost target and energy independence of both the deuteron break-up cross sections and the tensor analyzing power  $T_{20}$  in a large interval of energies [1,2,6]. Therefore, the deuteron break-up may be represented by a Feynman diagram in IA as shown in Fig. 1a. Due to the target independence of the cross section, it is intuitively clear that the vertices of the diagram should factorize and consequently they may be computed separately. In the Fig. 1b the target part of the cross section is depicted, and we begin our calculations with this subdiagram.

The corresponding amplitude is written in the form

$$A_{NN \rightarrow X} = \left( \bar{u}_X \Gamma_{NN \rightarrow X} \right)_{\alpha\beta} \left( u(p_p, s_p) \right)_\alpha \times \left( u(p_n, s_n) \right)_\beta, \quad (1)$$

where  $\left( \bar{u}_X \Gamma_{NN \rightarrow X} \right)$  is the truncated  $NN \rightarrow X$  vertex, and  $u(p, s)$  are the Dirac spinors for the incident nucleons. Then the cross section is given by

$$d\sigma = \frac{1}{16\sqrt{\lambda(p_p, p_n)}} \Lambda_{\alpha\alpha'}(p_p) \Lambda_{\beta\beta'}(p_n) O_{\alpha\alpha' \beta\beta'}, \quad (2)$$

where  $\lambda(p_1, p_2) = (p_1 p_2)^2 - m_1^2 m_2^2$  is the flux factor, and  $\Lambda(p)$  stands for the projection operator of an on-shell nucleon. The operator  $O$ , acting in the nucleon spinor space, is defined as

$$O_{\alpha\alpha'\beta\beta'} = \sum_X (2\pi)^4 \delta^4(p_p + p_n - P_X) \times \left[ \left( \bar{\Gamma}_{NN \rightarrow X} \mathcal{U}_X \right)_{\alpha'\beta'} \left( \bar{\mathcal{U}}_X \Gamma_{NN \rightarrow X} \right)_{\alpha\beta} \right] \times \frac{d^3 P_X}{2E_X (2\pi)^3}. \quad (3)$$

To evaluate this operator one needs, in principle, an analysis of the  $NN \rightarrow X$  vertices by expanding them in a basis of a complete set of  $(4 \times 4)$  matrices in the spinor space and a further partial wave analysis of the corresponding coefficients. For the elastic  $NN$  scattering such a procedure has been realized in Refs. [9,10]. At intermediate energies the spin-independent part of the amplitude dominates and the corrections from the spin-flip part are in the order of  $\approx 15\%$  [11]. In our present calculations the approximations, caused by neglecting the spin dependence of the amplitude, are partially compensated by the fact that we take the total cross section  $\sigma_{\text{tot}}$  from experiment (which contains all contributions). In addition, we intend to estimate the role of relativistic effects in the considered reactions by comparing our results with previously commonly accepted approaches. Therefore, in our case, and for the considered energies, one may neglect the spin structure of the  $NN$  vertices and take the operator (3) in its simplest form, i.e.,

$$O_{\alpha\alpha'\beta\beta'} \approx \delta_{\alpha\alpha'} \delta_{\beta\beta'} \cdot \mathcal{M}(p_p, p_n), \quad (4)$$

where  $\mathcal{M}(p_p, p_n)$  is a scalar function depending only on the initial energy of the nucleons.

Then the expression for the total  $NN$  cross section takes the form

$$\sigma_{\text{tot}} = \frac{1}{16\sqrt{\lambda(p_p, p_n)}} \text{Tr}(\hat{p}_p + m) \text{Tr}(\hat{p}_n + m) \times \mathcal{M}(p_p, p_n) = \frac{m^2}{\sqrt{\lambda(p_p, p_n)}} \mathcal{M}(p_p, p_n) \quad (5)$$

(the hat means contraction with Dirac's  $\gamma$  matrices).

The kinematics of the  $Dp \rightarrow pX$  reaction is defined according to the graph in Fig. 1a by  $p'_p = p_2, p_n = p_1,$

$p_{1,2} = \frac{1}{2}P_D \pm p$ , where  $p$  is the relative momentum and  $P_D$  is the total momentum of the deuteron. Then the amplitude of the basic process reads

$$A_{Dp \rightarrow pX} = \left( \bar{\mathcal{U}}_X \Gamma_{NN \rightarrow X} \right)_{\alpha\beta} (\bar{u}(p_2, s_2))_\gamma \times \left( \frac{\hat{p}_1 + m}{p_1^2 - m^2} \right)_{\beta\delta} (u(p, s_p))_\alpha (\Gamma_{D \rightarrow NN})_{\delta\gamma}, \quad (6)$$

where the truncated  $\Gamma_{D \rightarrow NN}$  vertex satisfies the BS equation. The cross section for the break-up of a polarized deuteron with a given spin projection  $M$  is

$$d\sigma_{Dp \rightarrow pX} = \sigma_{\text{tot}}(p_p, p_1) \frac{\sqrt{\lambda(p_p, p_n)}}{\sqrt{\lambda(p_p, P_D)}} \frac{1}{2m} \times \text{Tr} \left( \bar{\Psi}_M^D(p) \hat{f} \Psi_M^D(p) (\hat{p}_2 - m) \right) \times \frac{(p_2^2 - m^2)}{2E_2} \frac{d^3 p_2}{(2\pi)^3}. \quad (7)$$

In deriving Eq. (7) we made use of Eq. (4), and instead of the truncated  $\Gamma_{D \rightarrow NN}$  vertex we introduced a modified BS amplitude,  $\Psi_M^D(p)$ , which includes the nucleon propagators in its definition [12]. As a consequence, in Eq. (7) the term  $(p_2^2 - m^2)$ , which is zero when the nucleon is on mass shell, appears explicitly. It cancels singularities in the BS amplitude induced by the propagators and ensures the expression (7) to be finite. The trace in Eq. (7) is evaluated by an algebraic formula manipulation code.

With this at hand it is easy to calculate the tensor analyzing power  $T_{20}$  in terms of cross sections (7) from the relation

$$\sqrt{2} T_{20} = \frac{d\sigma(M=1) + d\sigma(M=-1) - 2d\sigma(M=0)}{\sum_M d\sigma(M)} = 1 - \frac{d\sigma(M=0)}{\frac{1}{3} \sum_M d\sigma(M)}. \quad (8)$$

To determine the polarization transfer  $\kappa$  one needs to considerate reactions of the type  $D + p = p' + X$  with polarized final protons. In this case the corresponding expression for the cross section only slightly differs from the Eq. (7), namely, the unit matrix  $\hat{f}$  in Eq. (7) must be replaced by  $(1 + \gamma_5 \hat{s})/2$ , where  $\hat{s}$  is the contracted polarization four-vector of the outgoing proton. Experimentally the quantization axis for

this process is chosen in the direction vertical to the beam so that  $s = (0, \mathbf{P}_p)$ , and the polarization transfer  $\kappa \equiv (\mathbf{P}_p \mathbf{n}) / (\mathbf{P}_d \mathbf{n})$  (here  $\mathbf{n}$  is a unit vector perpendicular to the reaction plane) may be computed as  $\kappa = (d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)) / d\sigma_{\text{unpol}}$ , where  $\uparrow\uparrow$  ( $\uparrow\downarrow$ ) means the orientation of the deuteron spin and final proton spin with respect to the quantization axis.

### 3. Results and discussions

In the calculations we use our recent solutions [13] of the BS equation with a kernel with  $\pi, \omega, \rho, \sigma, \eta, \delta$  exchanges (parameters as in Table 1 in Ref. [13]). In obtaining numerical solutions of the BS equation we expanded the amplitude  $\Psi^D$  on the complete set of the Dirac matrices and solved a system of eight integral equations for partial amplitudes in the Wick-rotated system (for further details see Ref. [12]). It is worth mentioning that our partial amplitudes depend on both  $p$  and  $ip_0$ , whereas in the considered reactions with one nucleon on mass shell, the relative energy  $p_0$  is fixed by kinematics and it is real. Hence, one needs a numerical recipe for an analytical continuation of the amplitudes from the imaginary to real axis of relative energy. This procedure has been investigated by Tjon [7] in studying diagrams similar to that in Fig. 1a with the result that there is a small difference (less than 5%) between the analytically continued partial amplitudes at the appropriate  $p_0$  and simply putting  $p_0 = 0$ . In our calculation we follow the results of Ref. [7] and choose the relative energy  $p_0 = 0$ . (To estimate the error in the present case we have expanded our amplitudes in powers of  $p_0$  around  $p_0 = 0$ ; explicit calculations show that up to quadratic terms at  $p \rightarrow 0$  [1 GeV/c] the corrections amount 0.1% [12%].)

In Figs. 2 and 3 we present the results of our relativistic calculation of the cross section (7) and the deuteron tensor analyzing power  $T_{20}$  (8) and the polarization transfer  $\kappa$ . A comparison is also given with non-relativistic results which utilize the Bonn wave function [14] and with results of the minimal relativization of the Bonn and Paris wave functions within the light cone dynamics [3,4]. Experimental data are from Refs. [1,2,6,15]. It is seen that our calculated values of the cross section and  $T_{20}, \kappa$  coincide with the non-relativistic calculations up to  $p' \sim 0.25$  GeV/c. Differences occur for larger proton momenta. For  $T_{20}$

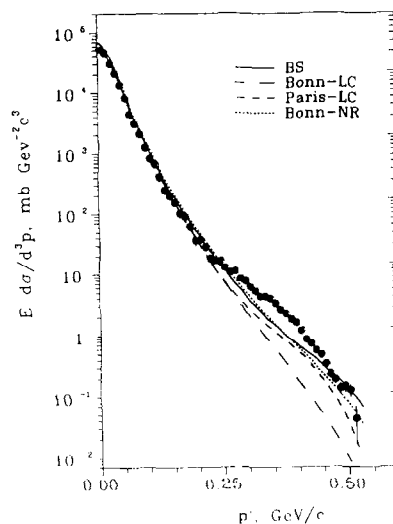


Fig. 2. The differential break-up cross section as function of the final proton momentum  $p'$  in the deuteron rest frame (solid line – our relativistic calculation within the Bethe-Salpeter formalism, dotted line – non-relativistic calculation with the Bonn deuteron wave function, long-dashed (short-dashed) line – results of a minimal relativization within the light cone dynamics of the Bonn (Paris) wave functions). Experimental data from Ref. [1].

these differences can be traced back to admixtures of negative energy states in the partial BS amplitudes, in particular the  $P$ -waves in the deuteron. The relative weight of the  $P$ -waves is rather small and in the unpolarized case the contribution of these negative energy states is negligible [7,8]. However, in the polarization case they play a more important role and lead to an improvement of the description of the data. As seen in Fig. 3 (left panel), even a consistent relativistic calculation does not yet describe satisfactorily the data in the region of the minimum of  $T_{20}$  at  $p' \sim 0.3$  GeV/c. Note that this region is just the one where the unpolarized cross section has a broad shoulder, see Fig. 2. As mentioned above, this shoulder may be reproduced by considering some other diagrams with final state interaction mediated by meson exchanges [5]. Other kinds of corrections, e.g., the rescattering effects have been computed in Ref. [2]. Both these mechanisms, meson exchanges and rescattering effects, reduce the depth of the minimum in Fig. 3 and lead to a shift of the sign change of  $T_{20}$  to larger values of  $p'$ . With this in mind one may expect that relativistic calculations, completed by corrections from the other mentioned mechanisms, describe both sets of experimen-

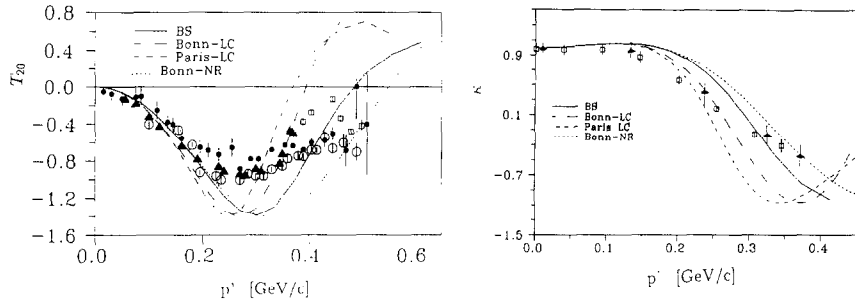


Fig. 3. The tensor analyzing power  $T_{20}$  (left panel) and the polarization transfer  $\kappa$  (right panel) as function of  $p'$  (the meaning of the curves is as in Fig. 2; experimental data on the left panel: triangles – Ref. [2], open circles – Ref. [6], open squares – Ref. [6] (elastic backward  $pD$  scattering), solid circles – Ref. [1] ( $^{12}\text{C}(D, p)X$  reactions); experimental data on the right panel: triangles – Ref. [15] ( $^{12}\text{C}(D, p)X$  reactions), open squares – Ref. [2]).

tal data in Figs. 2 and 3. The present difference between relativistically calculated values of  $T_{20}$  and the experimental data may be referred to as a signal of manifesting some unconventional mechanisms (like meson exchange currents,  $\Delta$  isobars, final state interaction, 3-body interaction, etc.) in the deuteron break-up reactions. It should be emphasized that our present approach seems best to describe the overall trend of the available data on the polarization transfer  $\kappa$  (see Fig. 3, right panel). However, also for this observable an improvement is needed.

The approach based on the minimal relativization scheme describes also rather well the differential cross section (see Fig. 2) but results in a more drastic disagreement with data for polarization observables (see Fig. 3). The minimal relativization procedure [3,4] consists of (i) a replacement of the argument of the non-relativistic wave functions by a light cone variable  $k^2 = m^2(1 - 2x)^2/4x(1 - x)$ , where  $x = (E_{p'} + p'_p)/(E_D + P_D)$  and (ii) multiplying the wave functions by a factor  $\sim 1/(1 - x)$ . As a result the argument is shifted towards larger values and the wave function itself decreases more rapidly. This effect of suppressing the wave function is compensated by the kinematical factor  $1/(1 - x)$ . Hence, the minimal relativization leads to a rather good description of spin averaged observables in the deuteron. A different situation occurs in case of computing polarization observables which are proportional to the ratio of cross sections. In this case all the kinematical factors cancel out and the effect of relativization comes only from the shift in arguments of the  $S$  and  $D$  components of the deuteron. Clearly, this leads to a squeezing effect

of  $T_{20}$  in comparison with the non-relativistic values, as seen in Fig. 3. In the BS amplitude the relativistic effects are of dynamical nature and are not reduced to a simple shift in arguments and, in addition to  $S$  and  $D$ -waves, it contains negative energy components, i.e.,  $P$ -waves which allow for a more refined analysis of the data.

An analysis of polarization observables in two, at first glance, different processes, such as the deep-inelastic scattering of polarized leptons off polarized deuterons and the polarized deuteron break-up, persuades us to a tight relation between the deep-inelastic structure functions  $b_{1,2}(x)$  (see Ref. [16] for definition) and the tensor analyzing power  $T_{20}$ . In deep-inelastic processes one considers reactions of inclusive scattering similar to the break-up reaction; the deuteron structure functions  $b_{1,2}(x)$  are related to the cross section by a relation similar to that in Eq. (8), except that  $d\sigma(M)$  is replaced by the deuteron structure function  $F_2^D(x, M)$  and that  $b_{1,2}(x)$  are not normalized to  $\sum_M F_2^D(x, M)$ . If we introduce new, normalized, deep-inelastic structure functions by

$$\begin{aligned} \tilde{b}_{1,2}(x) &= \frac{F_{1,2}^D(x, M=1) + F_{1,2}^D(x, M=-1) - 2F_{1,2}^D(x, M=0)}{\sum_M F_{1,2}^D(x, M)}, \quad (9) \end{aligned}$$

the correspondence between  $T_{20}$  and  $\tilde{b}_{1,2}(x)$  will be almost one to one. Consequently, the relativistic effects in these two functions are expected to be very similar. This assertion is partially confirmed by the present work and by the calculations performed in Ref. [13].

#### 4. Summary

In summary, we present an analysis of relativistic effects in the deuteron tensor analyzing power  $T_{20}$  and polarization transfer  $\kappa$  within the Bethe-Salpeter formalism with realistic interaction. The minimal relativization scheme is discussed and a comparison of the results of two approaches, i.e., the relativization within the light-cone dynamics and covariant BS formalism, with experimental data is given. For a perfect description of the experimental data one needs obviously processes beyond the impulse approximation.

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